

Bayes' rule

- 1) Three urns contain red and green marbles as follows: urn 1 has 3 green and 5 red, urn 2 has 9 green and 7 red, and urn 3 has 15 green and 17 red. An urn is selected at random and a marble is then chosen at random from this urn. Find the probability that
 - a) the marble chosen comes from urn 1 and is red.
 - b) a green marble is chosen.
 - c) urn 3 is chosen, given that a red marble is chosen.
- 2) Box 1 contains 14 rubies and 10 diamonds and box 2 contains 10 rubies and 2 diamonds. A fair die is rolled. If you roll a 1, you pick a gem from the first box. Otherwise, you pick a gem from box 2. Find the probability that
 - a) you roll a 1 and pick a ruby.
 - b) you pick a diamond.
 - c) you rolled a 1 given that you chose a ruby.
- 3) There are 3 drawers containing socks. The first drawer contains 5 red and 3 yellow socks. Drawer 2 contains 1 red and 3 yellow socks. Drawer 3 contains 3 red and 5 yellow socks. A drawer is chosen at random and a sock is selected from that drawer. Find the probability that
 - a) you pick drawer 2 and a red sock.
 - b) you pick a red sock.
 - c) you picked drawer 3, given that you picked a yellow sock.
- 4) Jar 1 contains 5 cherry and 2 orange candies, and jar 2 contains 4 cherry and 8 orange candies. A jar is chosen at random and a candy is drawn at random from it. Find the probability that
 - a) jar 1 and a cherry candy are chosen.
 - b) an orange candy is chosen.
 - c) jar 2 is chosen given that an orange candy is chosen.
- 5) Three bowls numbered 0, 1, and 2 contain red and blue chips. Bowl 0 contains 12 blue and 4 red, bowl 1 contains 14 blue and 2 red, and bowl 2 contains 4 blue and 28 red. A fair coin is tossed twice and a chip is drawn at random from the bowl whose number is the same as the number of heads obtained from tossing the coin. Find the probability that
 - a) a blue chip is chosen.
 - b) at most 1 head is tossed and a red chip is chosen.
 - c) the chip came from bowl 1, given that it is blue.
- 6) A box contains 3 green and 5 red balls. A second box contains 7 green and 2 red balls. A ball is randomly chosen from the first box and transferred to the second box without noting its color. Then a ball is randomly chosen from the second box. Find the probability that the transferred ball is red, given that the ball drawn from the second box is green.
- 7) On a game show, three boxes each contain 25 envelopes. In the first box 24 of the envelopes contain \$100. The second box contains 15 envelopes with \$100 and the third box has 12 envelopes that contain \$100. The rest of the envelopes are empty. A contestant rolls a fair die and selects an envelope at random from the first box if he rolls a 1, from the second box

if he rolls a 2 or 3, and from the third box if he rolls a 4, 5, or 6. Find the probability that the contestant chose an envelope from the first box, given that he won \$100.

8) A bag contains 2 pearls and nothing else. A fair die is rolled. If the outcome of the roll is 1 or 2, two white beads are added to the bag, if the outcome of the roll is a 3 or 4, four white beads are added to the bag, and if the roll is a 5 or 6, six white beads are added to the bag. Then an object is drawn at random from the bag. Given that a pearl was chosen, find the probability that 2 white beads were added to the bag.

9) A company has 3 machines, A, B, and C that all produce the same part. Machine A produces $\frac{1}{2}$ of the parts, machine B produces $\frac{1}{4}$ of the parts, and machine C produces $\frac{1}{4}$ of the parts. $\frac{3}{8}$ of the parts produced by A are defective, $\frac{1}{4}$ of those produced by B are defective, and $\frac{3}{4}$ of those produce by C are defective. Given that a part chosen at random is defective, find the probability that it was produced by machine C.

10) A blue cookie jar contains 3 chocolate, 4 almond, and 1 oatmeal cookie. A green jar contains 7 chocolate, 5 almond, and 4 oatmeal cookies. One of the jars is chosen at random and a cookie drawn from it. Given that the cookie chosen is chocolate, find the probability that it came from the blue jar.