MAT 3701: Assignment 7 Selected Solutions

Additional Problems

1. A bolt manufacturer knows that 5% of her production is defective. She gives a guarantee on a shipment of 10,000 parts by promising to refund the money if more than a bolts are defective. Based on Chebysev's inequality, how small can the manufacturer choose a and still expect to give a refund no more than 1% of the time? (In MAT 3702 you will learn how to get a much closer estimate using the normal approximation to the binomial distribution. And, of course, you can also use tables or statistical software to calculate the correct figure with any desired accuracy directly from the binomial distribution.) (Adapted from *Introduction to Probability Theory*, by Hoel, Port, and Stone, 1971, p. 107.)

   Solution I have chosen to focus on this problem because this type of reasoning is quite important practically; in addition, I plan to give a similar problem on the final.

   First, what type of random variable should we look at? Each bolt may be represented by a Bernoulli random variable: either it will be defective or not. We are counting the percentage of defective bolts, so let's set it up so 1 means defective and 0 means not defective. A shipment of $10,000$ bolts (presumably selected at random from the output of the factory) is then a binomial random variable, the sum of $10,000$ independent Bernoulli random variables with parameter $p = .05$ (since 5% of the production is defective). Thus, let X_i, $i = 1, 2, 3, \ldots, 10,000$ be independent Bernoulli random variable with parameter $p = .05$, and let $X = \frac{\sum_{i=1}^{10,000} X_i}{10,000}$. X then represents the percentage of actually defective bolts in the shipment.