MAT3701: Midterm Exam **Name:** ______________________________

March 6, 2013.

No notes or books are allowed. You may have writing implements, blank paper for scratch work, and a calculator (although you won't really need one).

Each numbered question has the same value, as do any lettered parts within a question.

1 Calculations

These questions will be graded 0-4 as follows: 0 = "nothing really there"; 1 = "a start"; 2 = "substantial (correct) progress"; 3 = "mostly correct"; 4 = "correct & complete (except possibly for trivial errors)."

1. (a) How many ways are there to partition a set of 7 distinct objects into subsets of size 2, 2, and 3? *Do not just write the symbol for the appropriate multinomial coefficient. Show how to calculate it. It is o.k. to use factorials in your answer, and you do not need to simplify it.*

An urn contains 3 red balls, 4 white balls, and 5 blue balls.

(b) 7 balls are drawn at random from the urn without replacement. What is the probability of the event E that 2 are red, 2 are white, and 3 are blue? *Leave the symbols for the appropriate binomial and/or multinomial coefficients in your answer.*

(c) A ball is drawn at random and replaced 7 times. What is the probability of the event E that 2 drawn balls are red, 2 are white, and 3 are blue? *Leave the symbols for the appropriate binomial and/or multinomial coefficients in your answer.*

2. Two fair six-sided (cubical) dice, each with faces numbered 1, 2, 3,4, 5, and 6, are tossed independently. One is white and the other is black. Let X be the number (facing up) on the white die, let Y be the number on the black die, and let Z be the sum of the two. In the problems that follow, recall that the *probability distribution* of a discrete random variable X is the function f defined by $f(x) = P(X = x)$; the probability distribution of a discrete random vector (X, Y) is the function f defined by $f(x, y) = P(X = x, Y = y)$.

   (a) Give a formula for the probability distribution of X.

   (b) Is the probability distribution for Y different from that of X? **Y** **N**

   (c) Give the joint probability distribution of X and Y; that is, give the probability distribution of the random vector (X, Y).

   (d) Give the probability distribution of Z.

3. A radioactive substance has a half-life of 2 seconds. Let T be the number of seconds after a specified initial instant at which a specified atom of this material decays. As shown in class, T has an exponential probability density:

$$f(t) = \begin{cases} ke^{-kt}, \text{ for } t \geq 0 \\ 0, \text{ for } t < 0 \end{cases},$$

where k is a constant.

(a) What is k? *Don't just write the answer; show how you calculated it.*

(b) What is the probability that the atom decays between 5 and 10 seconds after the initial instant? *For simplicity, you may leave your answer in terms of k.*

4. A test for a certain genetic marker gives a positive result 90% of the time if the gene is present and 5% of the time if it is not. It is know from genetic sequencing that the gene is present in 20% of those tested. What is the probability that a random subject who tests positive actually has the gene? *Either leave your answer as a fraction or round it to two decimal places.*

2 Theory and conceptual foundations

This question must be answered with a proof in (short) essay form and will be graded A-F (4–0) according to the usual standards.

5. Let (S, Σ, ρ) be a probability space; that is, let S be a sample space with probability measure ρ on a σ-algebra Σ of measurable subsets of S. Since ρ is a probability measure, we refer to the measurable subsets as *events*. If E is an event, denote the complement of E in S (in other words, the set of all outcomes in S that are *not* in event E) by E^c.

   (a) Let E and F be events. Prove that $F \setminus E = F \cap E^c$ is an event.

   (b) Prove that if $E \subseteq F$, then $\rho(E) \leq \rho(F)$.