MAT 3701: Probability **Name:** ______________________________

Final Exam: Take-home portion, due by 5 p.m. on Thursday, May 2, 2013

You are expected to work on this exam alone and to refrain from talking about the exam to anyone except the professor until the time and date when it is due. You may use your own notes and any published materials that you like. (Cite sources appropriately.)

Your signature below attests to a pledge that you have done the exam according to the above instructions. (Please attach this cover page to your solutions.)

Signature:______________________________

Solutions must be typeset in Tex.

1. (a) Let

$$f(x) = \begin{cases} \frac{c}{x}, & \text{for } x = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

where $c \in \mathbb{R}$ is a constant. Show that f is *not* the density of any random variable for any value of c. (Hint: Use the fact from calculus that a certain series does not converge.)

(b) Let

$$f(x) = \begin{cases} \frac{c}{x(x+1)}, & \text{for } x = 1, 2, 3, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

- Show that f *is* a density for an appropriate value of c, and find that value. (Hint: You just need to choose c so that the sum of all the values is one. To do that, note that $\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$.
- Let X be a random variable with density f (where c has been appropriately chosen as above). Show that X does *not* have finite expectation.

2. A political analyst wants to determine the percentage of voters who plan to vote for a particular candidate for mayor of a medium-sized city (population about 500,000), let's call her Amanda. To this end, he plans to conduct a poll by randomly calling registered voters. (He does not care which of the other candidates they plan to vote for if not for Amanda, so he will simply ask if they plan to vote for Amanda or not.) Because the race is very close, the analyst wants to have 99% certainty that his estimate is accurate to within one percentage point. (In other words, he wants the probability that the percentage of voters who actually plan to vote for Amanda differs from the percentage in his sample by more than 1% to be less than 1%.) How many voters must he call?

You may solve this problem either using Chebyshev's inequality, as we discussed in class, or alternatively you may use the fact that a binomial distribution with more than about 30 trials is very closely approximated by a normal distribution with the same expectation and variance. You will find that a much smaller sample is needed than that indicated by Chebyshev's inequality, which is very rough. *Those who correctly solve the problem using the normal distribution will get one point extra credit!* This is a good way to get a leg up on MAT 3702.

You can find information about how to do the calculation using the normal distribution in Hoel, Port, and Stone (on reserve), and probably as well through readily available sources on the Web. Tables and calculators for the value of the normal distribution are readily available (such as in the back of our text or in the back of Hoel, Port, and Stone).