4.13.1. Conditional and Biconditional Deduction Problems

A. For each of the following symbolic arguments, show that the argument is valid by constructing a **deduction** of the argument.

1. $(P \rightarrow R)\ \therefore\ ((P \wedge Q) \rightarrow R)$

2. $(P \rightarrow Q)\ .\ (Q \rightarrow R)\ \therefore\ (P \rightarrow R)$

3. $((P \rightarrow Q) \rightarrow (R \rightarrow S))\ .\ (R \rightarrow T)\ .\ ((R \rightarrow T) \rightarrow (P \rightarrow Q))\ .$
$(Q \rightarrow (S \rightarrow U))\ .\ (T \rightarrow P)\ \therefore\ (R \rightarrow U)$

4. $(P \rightarrow \sim Q)\ .\ (R \rightarrow Q)\ .\ (\sim R \rightarrow \sim S)\ .\ (T \rightarrow S)\ \therefore\ (P \rightarrow \sim T)$

5. $(P \rightarrow R)\ .\ (Q \rightarrow R)\ \therefore\ ((P \vee Q) \rightarrow R)$

6. $(R \rightarrow (P \vee Q))\ .\ (Q \rightarrow P)\ \therefore\ (\sim P \rightarrow \sim R)$

7. $((P \vee Q) \rightarrow R)\ .\ (R \rightarrow S)\ .\ (T \rightarrow Q)\ .\ \sim S\ \therefore\ \sim T$

8. $(R \rightarrow (P \wedge Q))\ \therefore\ (\sim P \rightarrow \sim R)$

9. $(R \rightarrow (P \vee Q))\ .\ (P \rightarrow S)\ .\ ((S \vee T) \rightarrow Q)\ .\ \sim Q\ \therefore\ \sim(R \vee T)$

10. $((P \wedge Q) \rightarrow R)\ .\ (\sim S \rightarrow (Q \wedge \sim R))\ \therefore\ (P \rightarrow S)$

B. **Translate** each of the following English arguments into the formal language of Chapter Four, then show that the argument is valid by constructing a **deduction** of it.

1. If the chef is the killer then Nick will catch him in a lie, assuming Nora joins the conversation. Provided that Nick will catch the chef in a lie if the chef is the killer, the chef will confess to the crime. The chef will confess to the crime only if he's the killer. Therefore, if Nora joins the conversation Nick will catch the chef in a lie.

2. That consonantal segment is prevocalic if it occurs initially; otherwise it's voiceless. Provided it's either prevocalic or voiceless, it's both continuant and strident. Assuming it's continuant, it's tense if it's strident. If it's tense, then if it occurs initially it's palatalized. Therefore, that consonantal segment is palatalized and voiceless.

(*Adapted from Partee, ter Meulen and Wall 1990: 134, Problem 10e*)

3. The president will sign an executive order if the bill stalls in either the House or the Senate. The Widget lobby will mobilize only if the bill stalls in the Senate. Assuming Gizmo PAC holds a phone campaign, the bill will stall in the House. If Gizmo PAC doesn't hold a phone campaign, the Widget lobby will mobilize. Therefore, the president will sign an executive order.

4. If neither the butler nor the chauffeur killed the baron, then the cook did. The cook killed the baron if and only if the stew was poisoned. The chauffeur killed the baron just in case there was a bomb in the car. The stew wasn't poisoned, and the butler didn't kill the baron. Therefore, there was a bomb in the car.

(*Adapted from Partee, ter Meulen and Wall 1990: 134, Problem 10a*)

5. Either Neko is a cat who can't stop eating, or Jack is a cat who's been stealing Neko's food. Neko can stop eating if Jack hasn't been stealing her food. Neko is a cat if and only if Jack is. Therefore, Jack is a cat who's been stealing Neko's food.

6. If God exists, then He is omnipotent. If God exists, then He is omniscient. If God exists, then He is benevolent. If God can prevent evil, then if He knows that evil exists, then He is not benevolent if He does not prevent it. If God is omnipotent, then He can prevent evil. If God is omniscient, then He knows that evil exists if it does indeed exist. Evil does not exist if God prevents it. Evil exists. Therefore, God does not exist.

(*from Kalish and Montague 1980: 35, Problem 35*)

C. Show that each of the following sentences is a theorem, by constructing a **proof** of the sentence.

T1. $(P \rightarrow P)$

T2. $(\sim P \rightarrow (P \rightarrow Q))$

T3. $(Q \rightarrow (P \rightarrow Q))$

T4. $((\sim P \rightarrow P) \rightarrow P)$

T5. $((P \rightarrow \sim P) \rightarrow \sim P)$

T6. $((P \rightarrow (P \rightarrow Q)) \rightarrow (P \rightarrow Q))$

T7. $((P \rightarrow Q) \leftrightarrow (\sim Q \rightarrow \sim P))$

T7a. $((P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P))$

T7b. $((\sim Q \rightarrow \sim P) \rightarrow (P \rightarrow Q))$

T8. $((P \rightarrow (Q \rightarrow R) \leftrightarrow ((P \wedge Q) \rightarrow R))$

T8a. $((P \rightarrow (Q \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow R))$

T8b. $(((P \wedge Q) \rightarrow R) \rightarrow (P \rightarrow (Q \rightarrow R))$

T9. $((P \rightarrow Q) \leftrightarrow (\sim P \vee Q))$

T9a. $((P \rightarrow Q) \rightarrow (\sim P \vee Q))$

T9b. $((\sim P \vee Q) \rightarrow (P \rightarrow Q))$

T10. $((P \to Q) \leftrightarrow \sim(P \wedge \sim Q))$

T10a. $((P \to Q) \to \sim(P \wedge \sim Q))$

T10b. $(\sim(P \wedge \sim Q) \to (P \to Q))$

T11. $((P \vee Q) \leftrightarrow ((Q \to P) \to P))$

T11a. $((P \vee Q) \to ((Q \to P) \to P))$

T11b. $(((Q \to P) \to P) \to (P \vee Q))$

T12. $(((P \vee Q) \to R) \leftrightarrow ((P \to R) \wedge (Q \to R)))$

T12a. $(((P \vee Q) \to R) \to ((P \to R) \wedge (Q \to R)))$

T12b. $(((P \to R) \wedge (Q \to R)) \to ((P \vee Q) \to R))$

T13. $(\sim(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q))$

T13a. $(\sim(P \leftrightarrow Q) \to (P \leftrightarrow \sim Q))$

T13b. $((P \leftrightarrow \sim Q) \to \sim(P \leftrightarrow Q))$

T14. $((P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow (P \wedge Q)))$

T14a. $((P \leftrightarrow Q) \to (P \leftrightarrow (P \wedge Q)))$

T14b. $((P \leftrightarrow (P \wedge Q)) \to \sim(P \leftrightarrow Q))$

D. **Derived Rule Problems**: For each of the following deductive systems, show that one or more of our deductive rules are derived rules in that system (by constructing a deduction of that rule, in that system).

1. The deductive system **MIN 1** is like our Chapter Four deductive system, except that it lacks the rules **R**, **~–**, and **MP**.

1a. Show that **R** is a derived rule in MIN 1.

1b. Show that **~–** is a derived rule in MIN 1.

1c. Show that **MP** is a derived rule in MIN 1.

2. The deductive system **MIN 2** is like our Chapter Four deductive system, except that it lacks the rules **R**, **~–**, and **~+**, and it also lacks **ID**. In its place, MIN 2 has the following rule, called "Negated Conditional" (or "~**→**").

(Any ID from our system of deduction can be converted into a CD in MIN 2, supplemented by **~→.)**

2a. Show that R is a derived rule in MIN 2.

2b. Show that ~– is a derived rule in MIN 2.

2c. Show that ~+ is a derived rule in MIN 2.