4.2. Conditional Clues and Complications

1. Antecedent and Consequent. We will find the conditional pickier and trickier than earlier molecular sentences, calling for distinctions between its parts that the other sentences did without. For this reason we distinguish between what we might, for the moment, call the '*if' part* of the conditional and its *'then' part*. And since these names will not always prove appropriate, we coin two new bits of logical jargon: the (so-called) 'if' part is the **antecedent** of the conditional, while the 'then' part is the **consequent**. In our earlier example, "Rex's team lost" was the antecedent, and "Rex is upset" was the consequent.

If Rex's team lost, then Rex is upset.

Antecedent **Consequent**

While antecedent and consequent in this English sentence are each marked by a special word ("if" and "then," respectively), Formalese instead marks them by their location: the **antecedent** of a formal conditional always comes **before the arrow**, the **consequent** always **after the arrow**. In the following formal conditional the location of the parts is sufficient to mark "P" as the antecedent and "Q" as the consequent.

P: Rex's team lost
Q: Rex is upset

If Rex's team lost, **then** Rex is upset

(P → Q)

Such terminology marks a departure from our more casual attitude in discussing conjunctions and disjunctions; for with those earlier sentences we needed no label fancier than 'left part' and 'right part'. That was due to the **commutativity** of the wedge and vel: so far as truth and validity go, the order of parts makes no difference in a conjunction or disjunction. Whenever it's true that "We're having ice cream and we're having cake," it's true that "We're having cake and we're having ice cream" (and vice versa); and likewise whenever it's true that "Either we're having cake or

we're having ice cream," it's true that "Either we're having ice cream or we're having cake" (and vice versa). With order of parts making no difference to truth or validity, we were indifferent to which part was which – and our generic labels for these parts reflected that.

But which part is which does make a difference to a conditional, since swapping antecedent and consequent can change a true conditional into a false one (or a false into a true one). Even if it is **true** that

> **If** Rex's team lost, **then** Rex is upset

it may well be **false** that

> **If** Rex is upset, **then** Rex's team lost.

Perhaps lots of things make Rex upset, and here it's someone keying his car, or the dogs getting into the garbage. In general: **switching antecedent and consequent in a true conditional is *not* guaranteed to yield another true conditional**.

That has immediate consequences for translation. Unless we are painstaking about isolating the antecedent and consequent in English, and placing each in its proper spot in the Formalese conditional, we may translate a true English sentence into a false formal one.

While that may illustrate how conditionals are *picky* – about which part is which – it doesn't make them look especially *tricky*. After all, the antecedent of an "if... then" sentence is marked as such with "if," and the consequent by "then"; so keeping them straight looks simple.

There's a second clue here anyway: the order of parts in an "if... then" sentence perfectly parallels the formal language: both languages place the antecedent first, and consequent last. With these two clues in hand, translating English conditionals into formal ones seems trouble-free.

But that overlooks two complications of English, both familiar from the previous chapter. Taking note of these, we will find that neither of the above clues are reliable markers of antecedent and consequent. And

appreciating that point, we will appreciate as well how translating conditionals can indeed be *tricky*.

2. First Complication: Translation Variations. In Chapter Three we found that every model example of logical form in English – "and," "or," and "not" – came with a variety of cousins meaning the same thing, and translated into Formalese the same way: the "**translation variations**" on these model cases.

English conditionals are no exception. All the following phrases count as **conditional phrases** of English, translated by an arrow.[1]

Ordinary Conditional Phrases:
If P then Q
If P, Q
Provided (that) P, Q
Assuming (that) P, Q
On the condition that P, Q

Exceptional Conditional Phrases ("Only" Phrases):
P only if Q
P only on the condition that Q

(Why we divide the list into two groups is explained below.)

So the following conditionals are translated into the same formal sentence.

P: It's raining **Q**: It's cloudy

If it's raining, **then** it's cloudy
If it's raining, it's cloudy
Provided that it's raining, it's cloudy
Assuming that it's raining, it's cloudy
On the condition that it's raining, it's cloudy

It's raining **only if** it's cloudy
It's raining **only on the condition that** it's cloudy

} **(P → Q)**

[1]Adapting the translation variations in Kalish and Montague 1964: 11 and 1980: 13, Problem 13.

As the first two examples illustrate, "if" can appear with or without its partner "then" – revealing "then" as a purely optional part (like optional "both" with "and," and "either" with "or").

We see now why we traded in the phrases "'if' part" and "'then' part": not all English conditionals contain the words "if" and "then". But every conditional has an antecedent and a consequent.

We see as well why our first proposed clue for finding antecedent and consequent is not, after all, reliable: since not every English conditional contains "if" and "then," we can't count on the antecedent to be flagged by "if," nor the consequent by "then".

3. Second Complication: Inversion. A further complication comes from inverted English sentences. The 'standard' English conditional

Antecedent Consequent
If **it's raining**, **it's cloudy**

can be inverted to become

Consequent Antecedent
It's cloudy if **it's raining**.

Inversion is familiar from conjunctions and disjunctions: the 'standard' disjunction "We'll have a picnic unless it rains" can be inverted to become "Unless it rains, we'll have a picnic." With disjunctions and conjunctions inversion could be taken in stride, since the order of the parts made no difference to truth (or validity). But we can't be so casual about the order of parts in a conditional: for as noted earlier, "$(P \rightarrow Q)$" may be true while "$(Q \rightarrow P)$" is false.

And this makes inversion a particularly unwelcome complication when translating conditionals. For now the second proposed clue in distinguishing antecedent and consequent – that antecedent comes first in English,

consequent after – *also* proves unreliable. Note that both the standard and inverted conditionals here have the same antecedent ("it's raining") and consequent ("it's cloudy").

Standard (antecedent first):	**Inverted** (consequent first):
If it's raining, it's cloudy	It's cloudy, if it's raining

Thanks to translation variations, we can't trust English to mark the antecedent with "if" and consequent with "then". And thanks to inversion, we can't trust English to put the antecedent first. Still, proper translation requires us to tell which part is antecedent, which consequent.

With both our earlier clues knocked out, translating conditional from English to the formal language looks practically impossible.

4. A General Rule for Translating Conditionals. But tucked in our example of inversion is a simple clue. Note that in the examples above, the **conditional phrase** – "if" – **comes right before the antecedent**, in both standard and inverted conditionals.

Standard (antecedent first):	Inverted (consequent first):
If it's raining, it's cloudy	It's cloudy, **if** it's raining

This holds for ordinary conditional phrases in general: whether the conditional is standard or inverted, an **ordinary conditional phrase** comes **right before the antecedent**.

Standard Conditional	Inverted Conditional
Ordinary Conditional Phrases (Before Antecedent) ***If*** it's raining, ***then*** it's cloudy ***If*** it's raining, it's cloudy ***Provided (that)*** it's raining, it's cloudy ***Assuming (that)*** it's raining, it's cloudy ***On the condition that*** it's raining, it's cloudy **"Only" Phrases** (Before Consequent) It's raining ***only if*** it's cloudy It's raining ***only on the condition that*** it's cloudy	**Ordinary Conditional Phrases** (Before Antecedent) [no inverted form][2] It's cloudy ***if*** it's raining It's cloudy ***provided (that)*** it's raining It's cloudy, ***assuming (that)*** it's raining It's cloudy ***on the condition (that)*** it's raining **"Only" Phrases** (Before Consequent) ***Only if*** it's cloudy is it raining ***Only on the condition that*** it's cloudy is it raining

And now it's clear why we group the "only" phrases separately: they are exceptional, since **"only" phrases come right before the consequent**.

This slim clue will prove sufficient for carving English language conditionals at their joints, and identifying antecedent and consequent.

[2] Recognizing "if... then" as the one English conditional phrase which *cannot* be inverted, we understand why, when "if... then" was our only example of a conditional phrase, it appeared that the antecedent would always come first: if the only conditional phrase of English were "if... then," there would be no inverted conditionals. The first complication (translation variations) brought the second (inversion) with it.

5. **More Complex Conditional Sentences: "Otherwise"**. In closing we note a couple of more complex sorts of sentences involving conditionals – as in the following example.

> **If the weather is nice then we'll hold Logic Fest in the park; otherwise we'll hold it in the pavilion.**
>
> **P**: The weather is nice
> **Q**: We'll hold Logic Fest in the park
> **R**: We'll hold Logic Fest in the pavilion
>
> **If P then Q; otherwise R**.

We know already to translate "If P then Q" as "$(P \rightarrow Q)$".

> $(P \rightarrow Q)$; otherwise R

The word "otherwise" here means: "if not P"; so "**otherwise R**" is translated as a second conditional, "$(\sim P \rightarrow R)$". The whole sentence is thus a **conjunction of two conditionals**.

> **If the weather is nice then we'll hold Logic Fest in the park; otherwise we'll hold it in the pavilion.**
>
> $$((P \rightarrow Q) \wedge (\sim P \rightarrow R))$$

We will revisit "other" as a kind of negation phrase in the more complex sentences of Chapter Six. [3]

[3] Some computers languages use the sentence form "If P then Q else R" which is equivalent to "If P then Q; otherwise R". As will appear in Chapter Six, "else" likewise acts a sort of negation phrase.

Summary

English Conditionals:

- **Ordinary** conditional phrases come right before the **antecedent**
- "**Only**" phrases come right before the **consequent**

Formal Conditionals:

- The **antecedent** goes **before** the arrow
- The **consequent** goes **after** the arrow

"Otherwise":

- "If P then Q; otherwise R" is translated as a conjunction of two conditionals.

  $((P \rightarrow Q) \wedge (\sim P \rightarrow R))$