4.14. Alternative Systems of Rules and Deduction

The idea of a derived rule of inference is already familiar from Chapter Three: a derived rule of inference is a convenient rule which is not one of the basic rules of a deductive system, but for which a deduction can be provided using only the basic rules of that system. De Morgan's Law was adopted as a derived rule in the Chapter Three deductive system. With the addition of further basic rules of inference in the Chapter Four deductive system, new possibilities for derived rules arise as well.

In particular: we will here explore alternative sets of rules (all derived rules in our deductive system) whose adoption as basic rules would allow us to treat current parts of our system as derived.

1. Eliminating Conditional Deduction. We also added a new form of deduction – Conditional Deduction (CD) – devoted only to deducing conditionals. But with the introduction of one or another further rule, we could always

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1. Eliminating ID. Add a new rule, **Non-Contradiction** (**NC**):

$$\frac{\quad}{\therefore \sim(\bullet \wedge \sim\bullet)}$$

Then replace each ID with a CD proving a sentence of this form:

If (AID), then $(\bullet \wedge \sim\bullet)$

2. Eliminating CD. Add a new rule, **Negated Conditional** (~→):

$$\frac{\sim(\bullet \rightarrow \blacktriangle)}{\therefore\ (\bullet \wedge \sim\blacktriangle)}$$

Then deduce conditionals by using ID.

(*Note that the converse rule*

$$\frac{(\bullet \wedge {\sim}\blacktriangle)}{\therefore\ {\sim}(\bullet \rightarrow \blacktriangle)}$$

is already deducible using ID and MP.)

3. Eliminating CD. Add a new rule, **Negation Disjunctiuon** (~∨).

$$\frac{({\sim}\bullet \vee \blacktriangle)}{\therefore\ (\bullet \rightarrow \blacktriangle)}$$

Then deduce a conditional by first getting its disjunctive counterpart (here, the premise) through ID.

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(*Note that the converse rule*

$$\frac{(\bullet \rightarrow \blacktriangle)}{\therefore\ ({\sim}\bullet \vee \blacktriangle)}$$

is already deducible using ID, DM, and MP.)

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Eliminating CD: An Example

1. $(P \rightarrow Q)$

2. $(Q \rightarrow R)$

Get: $(P \rightarrow R)$ (CD)

3.	P	ACD
4.	Q	1, 3, MP
5.	R	2, 4, MP
6.	$(P \rightarrow R)$	3, 5, CD

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1. $(P \rightarrow Q)$

2. $(Q \rightarrow R)$

Get: $(P \rightarrow R)$ (ID)

3.	$\sim(P \rightarrow R)$	AID
4.	$(P \wedge \sim R)$	3, $\sim\rightarrow$
5.	P	4, $\wedge-$
6.	Q	1, 5, MP
7.	R	2, 5, $\wedge+$
8.	$\sim R$	4, $\wedge-$
9.	$(P \rightarrow R)$	3, 7, 8, ID

Eliminating ID: An Example

1. $\sim P$

Get: $\sim(P \land Q)$ (ID)

2. $\sim\sim(P \land Q)$ AID
3. $(P \land Q)$ 1, $\sim -$
4. P 3, $\land -$
5. $\sim P$ 1, R
6. $\sim(P \land Q)$ 2, 4, 5, ID

1. $\sim P$

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Get: $(\sim\sim(P \land Q) \rightarrow (P \land \sim P))$ (CD)

2. $\sim\sim(P \land Q)$ ACD
3. $(P \land Q)$ 1, $\sim -$
4. P 3, $\land -$
5. $\sim P$ 1, R
6. $(P \land \sim P)$ 4, 5, $\land +$
7. $(\sim\sim(P \land Q) \rightarrow (P \land \sim P))$ 2, 6, CD
8. $\sim(P \land \sim P)$ NC
9. $\sim\sim\sim(P \land Q)$ 7, 8, MT
10. $\sim(P \land Q)$ 9, $\sim -$