❖ *Formal Semantics: Truth Tables and Truth Trees* ❖

4.3. Conditional Semantics

Our previous strategy for developing a formal semantics was to match each 'cyclical' (recursive) construction rule (for negations, conjunctions, and disjunctions) with a likewise 'cyclical' truth rule. Having introduced conditionals into the formal language by way of a new construction rule, we must now match that with a truth rule in order to maintain semantic coverage of the entire formal language.

The construction rule for conditionals was like those for conjunctions and disjunctions: the connective unites two formal sentences, and parentheses wrap the outside edges.

> 5. If ● and ▲ are formal sentences, then (● → ▲) is a formal sentence.

As always our own understanding of simple English sentences serves as our guide in semantics, since the arrow is intended to faithfully mirror English conditional phrases such as "if… then".

Consider a specific English example.

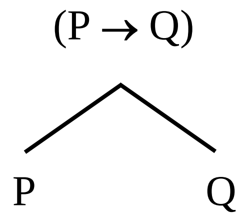
> **If Jack left before noon, then he avoided the rush-hour traffic**.

And translate this into formal language via the following translation table.

> **P**: Jack left before noon
> **Q**: Jack avoided the rush-hour traffic

> **(P → Q)**

The construction tree for "(P → Q)" is simple enough.



As usual, the semantics will repeat this construction, horizontally.

P	Q	(P → Q)

With two sentence letters, four valuations are called for.

4 =	**2** x **2**		
	P	Q	(P → Q)
1	1	1	
2	1	0	
3	0	1	
4	0	0	

We expect the truth of the conditional to depend on the truth of its parts: the antecedent "Jack left before noon," and the consequent "Jack avoided the rush-hour traffic". So where we disagree about whether the conditional holds true – perhaps to the point of wagering money on the matter – we settle the issue by asking Jack (i) whether he left before noon, and if so (ii) whether he avoided the rush-hour traffic.

The first case is straightforward: if it's true that *Jack left before noon*, and true that *Jack avoided the rush-hour traffic*, the whole conditional is true.

P	Q	(P → Q)
1	1	**1**
1	0	
0	1	
0	0	

The second valuation is equally simple: if it's true that *Jack left before noon*, but false that *Jack avoided the rush-hour traffic,* the conditional is false.

P	Q	$(P \rightarrow Q)$
1	1	1
1	0	**0**
0	1	
0	0	

The remaining two valuations are more perplexing, since they're both situations where it's **false** that *Jack left before noon* – yet the conditional only stakes a claim about what happened if he *did* leave before noon.

As bets go, we might call the whole thing off: no one won or lost. But formal semantics can't be so casual – for declaring in a similar way that the sentence is *neither true nor false* here would violate the Principle of Bivalence. Our semantic principles force us to take a stand.[1]

For a variety of reasons (discussed below) we declare the conditional **true** in cases **where its antecedent is false**.

P	Q	$(P \rightarrow Q)$
1	1	1
1	0	0
0	1	**1**
0	0	**1**

And we expect this to hold for conditionals in general, regardless of subject matter. So without reference to specific sentences such as "P" or "Q," we

[1] For a sentence to lack a truth value in a valuation is known technically as a "truth value gap" – something permitted in 'free logic,' but not in our classical semantics.

state the general semantic rule for conditionals: **a conditional is only false when its antecedent is true and its consequent false**.

Conditional Rule

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	1
0	0	1

The decision to make the conditional true whenever its antecedent is false – valuations 3 and 4 – might seem unintuitive on the face of it. But it can be defended by appeal to various judgments about **truth** and **sameness of meaning** which are themselves quite intuitive.

Were we not to count the conditional true in these controversial valuations, the only alternative Bivalence leaves is to have the conditional **false** in those cases.[2]

Pursuing that alternative, the semantics would yield the following rule for the conditional.

☠ Alternative Conditional Rule ☠

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	**0**
0	0	**0**

In that case a conditional would only be true when both its parts are true – precisely the semantic profile of conjunctions. Following this alternative rule would treat conditionals as **logically equivalent** to conjunctions.

[2] We consider a complication to this claim in the next reading, “*4.4. Conditional Semantics (Again): Converse, Contrapositive, and Biconditional*”.

But we take logical equivalence as evidence that two sentences **mean the same thing**. By consequence, this alternative conditional rule makes the bold semantic prediction that *a conditional and conjunction built from the same atomic sentences will mean the same thing*.

☠ Result of Alternative Conditional Rule ☠

P	Q	$(P \to Q)$	$(P \wedge Q)$
1	1	1	1
1	0	0	0
0	1	**0**	**0**
0	0	**0**	**0**

Is that right? Consider the following two subject matter sentences, and the conditional and conjunction constructed from them.

P: I won the lottery
Q: I'm a millionaire

Conjunction: "I won the lottery, **and** I'm a millionaire": $(P \wedge Q)$
Conditional: "**If** I won the lottery, **then** I'm a millionaire": $(P \to Q)$

As English speakers, we do **not** take these sentences to mean the same thing. So the alternative truth rule for the conditional makes an **incorrect prediction** about sameness of meaning.

(Actual facts illustrate vividly that the conjunction and conditional are not true in all the same cases: given the current lottery payoff, it's true that *if I won the lottery I'm a millionaire*; but as a sad matter of fact it's false that *I won the lottery and I'm a millionaire*.)

Having the conditional true whenever its antecedent is false permits our semantics to avoid such an incorrect prediction.

Our conditional rule makes further correct predictions about sameness of meaning. For instance, English speakers take the following pair of sentences to mean the same thing.

P: Rex will go out
Q: Rex will take his umbrella

“**It is not the case that** Rex will go out **without** taking his umbrella”
(“Rex wo**n’t** go out **without** taking his umbrella”): $\sim(P \wedge \sim Q)$

“**If** Rex goes out, he’ll take his umbrella”: $(P \rightarrow Q)$

Our semantic rule for conditionals judges these sentences to be logically equivalent – the correct result.

P	Q	~Q	$(P \wedge \sim Q)$	$\mathbf{\sim(P \wedge \sim Q)}$	$\mathbf{(P \rightarrow Q)}$
1	1	0	0	**1**	**1**
1	0	1	1	**0**	**0**
0	1	0	0	**1**	**1**
0	0	1	0	**1**	**1**

Likewise the following pairs of sentences mean the same thing; and our semantic rule for the conditional bears this out.

P: The picnic will be cancelled
Q: It rains

“The picnic won’t be cancelled unless it rains”: $\mathbf{(\sim P \vee Q)}$
“The picnic will be cancelled only if it rains”: $\mathbf{(P \rightarrow Q)}$

P	Q	~P	$\mathbf{(\sim P \vee Q)}$	$\mathbf{(P \rightarrow Q)}$
1	1	0	**1**	**1**
1	0	0	**0**	**0**
0	1	1	**1**	**1**
0	0	1	**1**	**1**

R: I'm mistaken
S: The quiz is on Thursday

"If I'm not mistaken, the quiz is on Thursday": **(~R → S)**
"Unless I'm mistaken, the quiz is on Thursday": **(R ∨ S)**

R	S	~R	**(~R → S)** [3]	**(R ∨ S)**
1	1	0	**1**	**1**
1	0	0	**1**	**1**
0	1	1	**1**	**1**
0	0	1	**0**	**0**

Additional evidence in favor of our conditional rule comes from **logical truth.**

The conditional "If it's raining, then it's raining" bears all the hallmarks of a logical truth (tautology): it's **always true** (whether it's raining or not), and seems to **convey no information** (when contemplating the merits of carrying an umbrella, a weather report that "If it's raining, then it's raining" is thoroughly uninformative). Truth tables bear this out.

P: It's raining

"If it's raining, then it's raining": **(P → P)**

●	▲	(● → ▲)
1	**1**	**1**
1	0	0
0	1	1
0	**0**	**1**

P	(P → P)
1	1
0	1

[3] Careful reading the truth table for "(~R → S)": only the fourth valuation makes the **antecedent** "~R" **true**, but the **consequent** "S" **false**.

Note that here again the alternative semantic rule for conditionals would get things badly wrong: since it judges the conditional *false* where antecedent and consequent are both false, that rule declares "(P → P)" false in the second valuation.

☠ Alternative Conditional Rule ☠

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	0
0	**0**	**0**

☠ Result of Alternative Conditional Rule ☠

P	(P → P)
1	1
0	**0**

But that's a mistake, for a couple of different reasons. First, even on a rainless day it's still true to say "If it's raining, then it's raining". So "(P → P)" should be true even when "P" is false. Second, this alternative truth table for "(P → P)" renders it **logically equivalent** to "P". Since we treat logical equivalence as a sign of sameness of meaning, this alternative truth table ends up claiming that "It's raining" means the same as "If it's raining, then it's raining". But that's wrong: the two sentences clearly mean quite different things; and the first is informative in a way the second is not.

In light of these and similar correct predictions – as well as later results concerning validity – we stand by our original truth rule for the conditional: **a conditional is only false when its antecedent is true, and its consequent is false**.

Formal Semantics: Chapter Four

1. Principle of Bivalence: In any possible situation, a sentence is either true or false (not both).

2. Negation Rule

●	~●
1	0
0	1

3. Conjunction Rule

●	▲	(● ∧ ▲)
1	1	1
1	0	0
0	1	0
0	0	0

4. Disjunction Rule

●	▲	(● ∨ ▲)
1	1	1
1	0	1
0	1	1
0	0	0

5. Conditional Rule

●	▲	(● → ▲)
1	1	1
1	0	0
0	1	1
0	0	1