❖ *Proofs and Deductions* ❖

4.12. Conditional Deduction

Indirect deduction introduced us to hypothetical reasoning: deducing what *would* happen, if a certain sentence *were* accepted as true. A similar strategy underlies a method of deduction devoted solely to conditional sentences.

Here, instead of hypothetically assuming the negation of the conclusion (as in ID), we assume the *antecedent* of the conditional sentence we're interested in deducing – exploring where that antecedent would lead. If we succeed in tracing a deductive trail from the assumed antecedent to the *consequent* of the conditional, we have shown that **if the antecedent were true, the consequent would be true**. Such hypothetical reasoning thereby establishes that the conditional is true.

For obvious reasons we call this new deductive strategy **Conditional Deduction** (or "**CD**" for short).

The following is an intuitively valid argument, with conditional conclusion.

> 1. We're having either ice cream or cake.
>
> ---
>
> (So,) If we're not having ice cream, then we're having cake.

Conditional Deduction provides a natural way of deducing this conditional conclusion from the premise.

We first restate the argument in formal language.

> **P**: We're having ice cream
> **Q**: We're having cake
>
> 1. $(P \vee Q)$
>
> ---
>
> $\therefore$ $(\sim P \rightarrow Q)$

The conditional deduction (CD) of the argument is set up like so.

1.	$(P \vee Q)$	
		Get: $(\sim P \rightarrow Q)$ (CD)

Then the hypothetical reasoning begins – as with ID, marked as hypothetical by occurring within a box.

First the **Assumption of the Conditional Deduction** (ACD): suppose we *did* have the antecedent.

1.	$(P \vee Q)$	
		Get: $(\sim P \rightarrow Q)$
2.	$\sim P$	ACD

From lines (1) and (2), "Q" follows by simple $\vee$–.

1.	$(P \vee Q)$	
		Get: $(\sim P \rightarrow Q)$ (CD)
2.	$\sim P$	ACD
3.	Q	1, 2, $\vee$–

But having demonstrated that if "~P" then "Q", we have established the truth of the whole conditional. The hypothetical reasoning is then complete.

1.	$(P \vee Q)$	
		~~Get~~: $(\sim P \rightarrow Q)$ (CD)
2.	$\sim P$	ACD
3.	Q	1, 2, $\vee$–
4.	$(\sim P \rightarrow Q)$	2, 3, CD

The conclusion of a conditional deduction is justified by citing the **assumption of the antecedent** (here, line 2), and the **consequent** deduced from that assumption (here, line 3).

Beyond their appeal to hypothetical reasoning, ID and CD are alike in other respects as well. First: as with IDs, once a CD box is closed all lines in that box become **unusable**. No rule can be applied to any line in a closed box.

And like IDs, CDs can be used **recursively**: in the midst of one CD, we can start another, hence embedding CDs within CDs. The following is a simple example.

1. We'll have either ice cream, or cake, or pie.

$\therefore$ If we don't have ice cream, then if we don't have cake we'll have pie.

1. $(P \vee (Q \vee R))$

$\therefore$ $(\sim P \rightarrow (\sim Q \rightarrow R))$

1.	$(P \lor (Q \lor R))$	
		~~Get~~: $(\sim P \rightarrow (\sim Q \rightarrow R))$ (CD)
2.	$\sim P$	ACD
3.	$(Q \lor R)$	1, 2, $\lor-$
		~~Get~~ $(\sim Q \rightarrow R)$ (CD)
4.	$\sim Q$	ACD
5.	R	3, 4, $\lor-$
6.	$(\sim Q \rightarrow R)$	3, 5, CD
7.	$(\sim P \rightarrow (\sim Q \rightarrow R))$	2, 6, CD

Conditional deduction is fundamentally unlike indirect deduction in one way, however: whereas ID is suitable for deducing *any* type of sentence, CD is only useful for deducing a conditional.

That point marks a change in our deductive strategy. While the advent of ID led to a default strategy of automatically reaching for ID, **for any conditional conclusion we now automatically use CD** (unless an easier way of getting that conclusion is obvious).

Summary: Conditional Deduction (CD)

- Write (**CD**) next to the "Get" line, as a reminder.
- Immediately following the "Get" line, begin a **box**, in which the Conditional Deduction occurs.
- The first line in the CD box is the **Assumption of the Conditional Deduction** (**ACD**): the **antecedent** of the sentence on the "Get" line.
- Using deductive rules on all available lines (premises and ACD), deduce the **consequent** of the sentence on the "Get" line.
- Once the consequent has been deduced, **close** the CD box. (When the CD box is closed, no rules can be applied to any line in that box. These sentences become "**unusable**".)
- Beneath the CD box write the conclusion of the argument (the sentence on the "Get" line). The justification for this conclusion cites two lines: the **ACD**, and the **consequent**. These two numbers are followed by "**CD**".

- If the conclusion of an argument is a **conditional**, automatically use **CD** for that argument. If the conclusion of the argument is any other type of sentence (sentence letter, negation, conjunction, or disjunction), use **ID**.