4.5.1. Conditional Semantic Problems:
Tautology, Contradiction, Logical Equivalence, and Validity

A. T**ranslate** each English sentence into formal and build a **truth table** for that sentence. On the basis of that truth table, find a **simpler English sentence** that is **logically equivalent** to the original.

1. If we either win the lottery or don't, we're going to Hawaii.

2. If we're going to Hawaii, then we're not going to Hawaii.

3. It's not the case that if we're having a quiz then we're not having one.

4. If Jake passed Chemistry, then if he passed Chemistry he'll graduate.

5. If we're having ice cream only if Suki won a prize, then we're having ice cream.

6. If we're having ice cream only if Suki won a prize, then Suki won a prize.

7. We'll have ice cream – or, if not, we'll have cake.

8. Neko went hungry if neither she nor Jack went hungry.

9. Neko went hungry only if neither she nor Jack went hungry.

10. Neko went hungry if and only if neither she nor Jack went hungry.

11. The tide is rising if, and only if, Jack is a pirate if and only if he's a pirate.

12. If Nick wants a drink, then he wants a drink if and only if Nora wants one.

(*For Problems 13 and 14, the simpler sentence won't appear as an earlier step in the truth table.*)

13. Suki won a prize if, and only if, we'll have champagne if she won a prize.

14. Suki won a prize if, and only if, we'll have champagne only if she won a prize.

(*For Problems 15 through 20, try to find a simpler* ***conditional*** *– it won't appear as an earlier step in the truth table.*)

15. Nick wants a drink if and only if both Nick and Nora want a drink.

16. Nora wants a drink if and only if either she or Nick wants a drink.

17. Either Nora wants a drink, or Nick wants a drink if and only if Nora wants one.

18. If we go to either Thailand or Hong Kong, we'll go to Hong Kong.

19. If we're having ice cream, we aren't having *both* ice cream *and* cake.

20. If Rex passed the exam, he did so without studying.

B. **Translate** each of the following sentences into formal language; then use a **truth table** or **truth tree(s)** to decide whether that sentence is a **tautology**, a **contradiction**, or **neither**.

1. If we’re going to Hawaii, then we’re going to Hawaii.

2. If we’re going to Hawaii, then we’re not going to Hawaii.

3. If we either win the lottery or don’t, we’re going to Hawaii.

4. If we’re going to Hawaii, then we’ll either win the lottery or we won’t.

5. If we win the lottery, then we’ll go to Hawaii without going to Hawaii.

6. If we either win the lottery or don’t, then we’ll go to Hawaii without going to Hawaii.

7. It’s raining if it’s raining; otherwise it’s not.

8. If it’s Tuesday, then if we have a quiz it’s Tuesday.

9. Rex took his umbrella if he went out, but he went out without taking his umbrella.

10. It’s raining if and only if it’s raining.

11. It’s not the case that: it’s raining if and only if it’s raining.

12. It’s raining if and only if it’s not raining.

13. It’s not the case that: it’s raining if and only if it’s not raining.

C. For each trio of sentences, build **truth tables** or **truth trees** to show which two sentences are **logically equivalent**. (For each trio, one of the sentences is *not* equivalent to the other two.)

1a. $((P \rightarrow Q) \rightarrow P)$
1b. $((P \rightarrow Q) \rightarrow Q)$
1c. $((P \rightarrow Q) \rightarrow (P \vee Q))$

2a. $((P \rightarrow R) \wedge (Q \rightarrow R))$
2b. $((P \wedge Q) \rightarrow R)$
2c. $((P \vee Q) \rightarrow R)$

3a. $((P \rightarrow R) \vee (Q \rightarrow R))$
3b. $((P \wedge Q) \rightarrow R)$
3c. $((P \vee Q) \rightarrow R)$

4a. $((P \rightarrow Q) \vee (P \rightarrow R))$
4b. $(P \rightarrow (Q \vee R))$
4c. $(P \rightarrow (Q \wedge R))$

5a. $(P \rightarrow (Q \rightarrow R))$
5b. $((P \rightarrow Q) \rightarrow R)$
5c. $((P \wedge Q) \rightarrow R)$

6a. $(P \rightarrow (Q \vee R))$
6b. $((\sim Q \vee \sim R) \rightarrow \sim P)$
6c. $(\sim(Q \vee R) \rightarrow \sim P)$

7a. $\sim(P \leftrightarrow Q)$
7b. $(\sim P \leftrightarrow Q)$
7c. $(\sim P \leftrightarrow \sim Q)$

D. Build a **truth table** or **truth tree** for each of the following sentences to show that the sentence is a **tautology**.

T1. $(P \to P)$
T2. $((P \to Q) \to ((Q \to R) \to (P \to R)))$
T3. $(P \to (\sim P \to Q))$
T4. $(P \to ((P \to Q) \to Q))$
T5. $(((P \to Q) \to P) \to P)$
T6. $((P \to Q) \vee (Q \to P))$
T7. $((P \to Q) \to ((P \vee R) \to (Q \vee R)))$
T8. $(((P \to Q) \wedge (R \to S)) \to ((P \wedge R) \to (Q \wedge S)))$
T9. $((P \to (Q \to R)) \leftrightarrow ((P \wedge Q) \to R))$
    T9a. $((P \to (Q \to R)) \to ((P \wedge Q) \to R))$
    T9b. $((P \to (Q \to R)) \to ((P \wedge Q) \to R))$
T10. $((P \to (Q \to R)) \leftrightarrow (Q \to (P \to R)))$
    T10a. $((P \to (Q \to R)) \to (Q \to (P \to R)))$
    T10b. $((Q \to (P \to R)) \to (P \to (Q \to R)))$
T11. $((P \to Q) \leftrightarrow (\sim P \vee Q))$
    T11a. $((P \to Q) \to (\sim P \vee Q))$
    T11b. $((\sim P \vee Q) \to (P \to Q))$
T12. $((P \to Q) \leftrightarrow \sim(P \wedge \sim Q))$
    T12a. $((P \to Q) \to \sim(P \wedge \sim Q))$
    T12b. $(\sim(P \wedge \sim Q) \to (P \to Q))$
T13. $((P \to \sim P) \leftrightarrow \sim P)$
    T13a. $((P \to \sim P) \to \sim P)$
    T13b. $(\sim P \to (P \to \sim P))$
T14. $((P \to (Q \wedge \sim Q)) \leftrightarrow \sim P)$
    T14a. $((P \to (Q \wedge \sim Q)) \to \sim P)$
    T14b. $(\sim P \to (P \to (Q \wedge \sim Q)))$

E. **Translate** each of the following arguments into formal language, then use **truth tables** or **truth trees** to decide if the argument is **valid**.

1. Provided Rex studied, he passed the exam. Assuming Rex passed the exam, he studied. ∴ Rex studied, and he passed the exam.

2. Either the patient is still in surgery, or Dr. Slim is having a drink. If the patient is still in surgery, Dr. Slim is having a drink. ∴ Dr. Slim is having a drink.

3. We'll spend the weekend in Chicago if the weather is good. We'll spend the weekend in Chicago if the weather isn't good. ∴ We'll spend the weekend in Chicago.

4. We'll spend the weekend in Chicago only if the weather is good. We'll spend the weekend in Chicago only if the weather isn't good. ∴ We won't spend the weekend in Chicago.

5. We'll have a picnic provided the weather is good; otherwise we won't. ∴ We'll have a picnic if and only if the weather is good.

6. Jake will get parole if the judge is lenient; otherwise he won't. ∴ Either the judge will be lenient and Jake will get parole, or the judge won't be lenient and Jake won't get parole.
   (*Hint: in a truth tree, start with the first premise*.)

7. Jack isn't a bird who can fly. ∴ If Jack is a bird, he's one that can't fly.
   (*Hint: see the remarks on relative clauses and negations in 3.10, Section 3*.)

8. Rex won't pass the exam without studying. ∴ Rex will pass the exam only if he studies.
   (*Hint: see the remarks on 'without' and negations in 3.10, Section 3*.)

9. If Nick knows who committed the crime then Nora does too. Nora doesn't know who committed the crime unless Nick knows. ∴ Both Nick and Nora know who committed the crime.

10. We're having a picnic only if it's sunny, assuming that we're having a picnic. We're not having a picnic if it's not sunny. ∴ It's sunny.

11. It's Thursday, assuming that if it's Thursday we're having a quiz. ∴ We're having a quiz, provided that it's Thursday only if we're having a quiz.

12. Assuming that William James is American only if Bertrand Russell is, Bertrand Russell is American. Bertrand Russell is American only if he's not American. ∴ Provided that Bertrand Russell is American if William James is, William James is American.

13. He's both trustworthy and responsible if he belongs to our group. He doesn't belong to our group. ∴ He's either untrustworthy or irresponsible.
(*from Kleene 1967/2002: 66, #14.1a*)

14. He's both trustworthy and responsible if and only if he belongs to our group. He doesn't belong to our group. ∴ He's either untrustworthy or irresponsible.

15. Jack will make a tuna salad if Neko goes skating; otherwise he'll bake a tuna casserole. Neko will go skating only if Jack will bake a tuna casserole. ∴ Neko will go skating and Jack will make a tuna salad.

16. Either Neko is a cat who can't stop eating, or Jack is a cat who's been stealing Neko's food. Neko can stop eating if Jack hasn't been stealing her food. If Neko is a cat, then so is Jack. ∴ Jack is a cat who's been stealing Neko's food.

17. The president will issue an executive order if the bill stalls in either the House or the Senate. The Widget lobby will mobilize only if the bill stalls in the Senate. Assuming Gizmo PAC holds a phone campaign, the bill will stall in the House. Provided Gizmo PAC doesn't hold a phone campaign, the Widget lobby will mobilize. ∴ The president will issue an executive order.

18. If God exists, then He is omnipotent. If God exists, then He is omniscient. If God exists, then He is benevolent. If God can prevent evil, then if He knows that evil exists, then He is not benevolent if He does not prevent it. If God is omnipotent, then He can prevent evil. If God is omniscient, then He knows that evil exists if and only if evil does exist. Evil does not exist if God prevents it. Evil exists. ∴ God does not exist.
(Adapted *from Kalish and Montague 1980: 35, Problem 35*)

19. If the butler didn't kill the baron, then either the cook or the chauffeur did. The stew was poisoned if the cook killed the baron, but there was a bomb in the car if the chauffeur killed him. The stew wasn't poisoned, and the butler didn't kill the baron. ∴ the chauffeur killed the baron.

(*Adapted from Partee, ter Meulen and Wall 1990: 134, Problem 10a. What's unusual about this argument?*)

20. If the chef is the killer then Nick will catch him in a lie, assuming Nora joins the conversation. Provided that Nick will catch the chef in a lie if the chef is the killer, the chef will confess to the crime. The chef will confess to the crime only if he's the killer. ∴ If Nora joins the conversation Nick will catch the chef in a lie.

21. That consonantal segment is prevocalic if it occurs initially; otherwise it's voiceless. Provided it's either prevocalic or voiceless, it's both continuant and strident. Assuming it's continuant, it's tense if it's strident. If it's tense, then if it occurs initially it's palatalized. ∴ That consonantal segment is palatalized and voiceless.

(*Adapted from Partee, ter Meulen and Wall 1990: 134, Problem 10e*)

22. If we have either ice cream or cake, then either we'll have ice cream without having pie or we'll have both brownies and sherbet. We'll have cake and brownies but we won't have both pie and fudge. Unless we have pie without having fudge, we'll have neither brownies nor sherbet. ∴ Either we'll have sherbet without having ice cream, or we'll have fudge without having ice cream.