3.5.1. Semantic Problems:
Tautology, Contradiction, Logical Equivalence, and Validity

A. **Translate** each English sentence into the formal language and build a **truth table** for that sentence. On the basis of that truth table, find a **simpler English sentence** that is **logically equivalent** to the original.

1. Rex got a royalty check if, and only if, he published if and only if he published.

2. Jack wore a helmet if he wore neither a helmet nor a spare parachute.

> (*For Problems 3 through 7, the simpler sentence won't appear as an earlier step in the truth table.*)

3. Assuming that Trixie won the poker game if and only if we'll have mochi, she won the poker game.

4. Neko went hungry only if neither she nor Suki went hungry.

5. Trixie won the poker game if, and only if, we'll have mochi if she won the poker game.

6. Trixie won the poker game if, and only if, we'll have mochi only if she won the poker game.

7. Neko went hungry if and only if neither she nor Suki went hungry.

(*For Problems 8 through 11, try to find a simpler* ***conditional*** *– it won’t appear as an earlier step in the truth table.*

For Problems 8 through 11, use the ***same translation key*** *for all four problems.*)

8. If Dick wants a martini, then he wants one if and only if Dora wants one too.

9. Dick wants a martini if and only if both he and Dora want a martini.

10. Dora wants a martini if and only if either she or Dick wants one.

11. Either Dora wants a martini, or Dick wants one if and only if Dora wants one.

B. **Translate** each of the following sentences into the formal language; then use a **truth table** or **truth trees** to decide whether that sentence is a **tautology**, a **contradiction**, or **neither**.

1. If Suki's going to Hawaii then she's going to Hawaii.

2. If Suki's going to Hawaii then she's not going to Hawaii.

3. If Suki either wins the lottery or doesn't, she's going to Hawaii.

4. If Suki's going to Hawaii, then she'll either win the lottery or she won't.

5. If Suki wins the lottery, then she's going to Hawaii without going to Hawaii.

6. If Suki either wins the lottery or doesn't, then she's going to Hawaii without going to Hawaii.

7. Rex taught Logic if he taught Logic; otherwise he didn't.

8. If Deacon's on time, then assuming he's on time he'll crack the safe.

9. If Deacon will crack the safe, then assuming he's on time he'll crack the safe.

10. Barbie took her umbrella if she went out, although she went out without taking her umbrella.

11. If Lucretia didn't skip class and she also passed the quiz, then she passed the quiz.[1]

12. If Lucretia didn't skip class and also pass the quiz, then she passed the quiz.[1]

11. Dr. Slim got sued if and only if he got sued.

12. It's not the case that: Dr. Slim got sued if and only if he got sued.

13. Dr. Slim got sued if and only if he didn't get sued.

14. It's not the case that: Dr. Slim got sued if and only if he didn't get sued.

15. Dr. Slim didn't get sued if and only if he didn't get sued.

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[1] See 2.9 § 4 on the effect deleted repetition has on connective scope.

C. **Translate** each of the following arguments into the formal language; then use **truth tables** or a **truth tree** to decide if the argument is **valid**.

1. If Suki's ticket is valid, then so is mine. My ticket's invalid. ∴ Neither Suki's ticket nor mine are valid.

2. If hyperbolic geometry is inconsistent, then so is Euclidean geometry. ∴ If Euclidean geometry is consistent, then so is hyperbolic geometry.

3. Provided she studied, Barbie passed Business Logic. Barbie passed Business Logic only if she studied. ∴ Barbie studied, and she passed Business Logic.

4. Either the patient is still in surgery, or Dr. Slim is having a drink. If the patient is still in surgery, Dr. Slim is having a drink. ∴ Dr. Slim is having a drink.

5. Dr. Slim is going to Reno if Kitty is, and Kitty's going to Reno. ∴ Both Kitty and Dr. Slim are going to Reno.

6. Trixie will play poker if Elvis does. Trixie will play poker (even) if Elvis doesn't. ∴ Trixie will play poker.

7. Trixie will play poker only if Elvis does. Trixie will play poker only if Elvis doesn't. ∴ Trixie won't play poker.

8. Either Suki will order sushi or Neko will. Neko will order sushi if Suki does. ∴ Both Suki and Neko will order sushi.

9. Either Suki will order sushi or Neko will. Neko will order sushi if and only if Suki does. ∴ Both Suki and Neko will order sushi.

10. If Dick knows who poisoned the gin then Dora does too. Dora doesn't know who poisoned the gin unless Dick knows. ∴ Both Dick and Dora know who poisoned the gin.

11. Jake is both trustworthy and responsible if he's a member of the Surf Club. Jake isn't a member of the Surf Club. ∴ Jake is either untrustworthy or irresponsible.

(*Adapted from* Kleene 1967/2002: 66, #14.1a)

12. Jake is both trustworthy and responsible if and only if he's a member of the Surf Club. Jake isn't a member of the Surf Club. ∴ Jake is either untrustworthy or irresponsible.

13. If Dr. Slim isn't a physician, he's not a physician who performs surgery. Dr. Slim isn't a physician. ∴ Dr. Slim doesn't perform surgery.

14. If Dr. Slim is a physician, he's one who doesn't perform surgery. Dr. Slim isn't a physician. ∴ Dr. Slim doesn't perform surgery.

15. Dick will have a Pimm's Cup if Dora has one; otherwise he won't. ∴ Either both Dick and Dora will have a Pimm's Cup, or neither of them will.

16. Lucretia went clubbing at Novo if she finished her lab report; otherwise she didn't. ∴ Lucretia went clubbing at Novo if and only if she finished her lab report.

17. Suki got an A if she passed the quiz; otherwise she got a B. Suki didn't get a B. ∴ Suki passed the quiz and got an A.

18. If Jack was arrested for scaling a skyscraper he's not running with the bulls in Pamplona; otherwise he is. ∴ Either Jack was arrested for scaling a skyscraper or he's running with the bulls in Pamplona, but not both.

19. Neko's not a bird that can fly. ∴ If Neko's a bird, she's one that can't fly.

(*Hint: see the remarks on relative clauses and negations in* 2.10, §3.)

20. Trixie won't pass Business Logic without studying. ∴ Trixie will pass Business Logic only if she studies.

(*Hint: see the remarks on '*without*' and negations in* 2.10, §3.)

21. Kitty has a kong of flowers only if she has a joker, assuming she has a kong of flowers. She doesn't have a kong of flowers if she doesn't have a joker. ∴ Kitty has a joker.

22. It's Thursday, assuming that if it's Thursday then Jack is surfing. ∴ Jack is surfing, provided that it's Thursday only if he's surfing.

23. Assuming William James is American only if Bertrand Russell is, Bertrand Russell is American. Bertrand Russell is American only if he's not American. ∴ Provided that Bertrand Russell is American if William James is, William James is American.

24. If Letitia liked the movie, it had a happy ending. If Lucretia liked the movie it didn't have a happy ending. ∴ Either Letitia liked the movie or Lucretia did, but not both.

25. If Letitia liked the movie, it had a happy ending. If Lucretia liked the movie it didn't have a happy ending. ∴ Letitia and Lucretia didn't both like the movie.

26. Kitty will have both a manicure and a massage if the check clears, and a manicure without a massage otherwise. ∴ Kitty will have a manicure, and she'll have a massage if and only if the check clears.

27. Either Neko is a cat who can't stop eating, or Jack is a cat who's been stealing Neko's food. Neko can stop eating if Jack hasn't been stealing her food. Assuming Neko is a cat, Jack's one too. ∴ Jack is a cat who's been stealing Neko's food.

28. The president will issue an executive order if the bill stalls in either the House or the Senate. The Widget lobby will mobilize only if the bill stalls in the Senate. Assuming Gizmo PAC holds a phone campaign, the bill will stall in the House. Provided Gizmo PAC doesn't hold a phone campaign, the Widget lobby will mobilize. ∴ The president will issue an executive order.

29. If God exists, then He's omnipotent, omniscient, and benevolent. If God is omniscient, then He knows that evil exists if and only if it does exist. If God is omnipotent, He can prevent evil. Provided God can prevent evil and knows that evil exists, He's not benevolent if He doesn't prevent it. Evil doesn't exist if God prevents it. Evil exists. ∴ God does not exist.

(*Adapted from* Kalish, Montague, and Mar 1980: 35, Problem 35)

30. If the bartender is the killer then Dick will catch her in a lie, assuming Dora joins the conversation. Provided that Dick will catch the bartender in a lie if the bartender is the killer, the bartender will confess to the crime. The bartender will confess to the crime only if she's the killer. ∴ If Dora joins the conversation, Dick will catch the bartender in a lie.

31. If the bartender didn't kill the baron, then either the sommelier or the bootlegger did. The merlot was poisoned if the sommelier killed the baron. There was antifreeze in the sour mash if the bootlegger killed the baron. The merlot wasn't poisoned, and the bartender didn't kill the baron. ∴ The bootlegger killed the baron.

> (*Adapted from* Partee, ter Meulen, and Wall 1990: 134, Problem 10a.
> *Q: What's unusual about this argument?*)

32. That consonantal segment is prevocalic if it occurs initially; otherwise it's voiceless. Provided it's either prevocalic or voiceless, it's both continuant and strident. Assuming it's continuant, it's tense if it's strident. If it's tense, then if it occurs initially it's palatalized. ∴ That consonantal segment is palatalized and voiceless.

> (*Adapted from* Partee, ter Meulen, and Wall 1990: 134, Problem 10e)

33. If we have either ice cream or cake, then either we'll have ice cream without having pie or we'll have both brownies and sherbet. We'll have cake and brownies but we won't have both pie and fudge. Unless we have pie without having fudge, we'll have neither brownies nor sherbet. ∴ Either we'll have sherbet without having ice cream, or we'll have fudge without having ice cream.[2]

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[2] This argument first appeared in 1.11. Warning: a truth table for this argument takes 1,625 steps (counting each sentence, 1, and 0 as a step).

D. Build a **truth table** or **truth tree** for each of the following sentences to show that the sentence is a **tautology**.

T3.1. $(P \rightarrow P)$

T3.2. $((P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R)))$

T3.3. $(P \rightarrow (\sim P \rightarrow Q))$

T3.4. $(P \rightarrow ((P \rightarrow Q) \rightarrow Q))$

T 3.5. $(((P \rightarrow Q) \rightarrow P) \leftrightarrow P)$

T 3.5a. $(((P \rightarrow Q) \rightarrow P) \rightarrow P)$

T 3.5b. $(P \rightarrow ((P \rightarrow Q) \rightarrow P))$

T 3.6. $(((P \rightarrow Q) \leftrightarrow (P \rightarrow (P \wedge Q))$

T 3.6a. $((P \rightarrow Q) \rightarrow (P \rightarrow (P \wedge Q)))$

T 3.6b. $((P \rightarrow (P \wedge Q)) \rightarrow (P \rightarrow Q))$

T3.7. $((P \rightarrow Q) \vee (Q \rightarrow P))$

T3.8. $((P \rightarrow Q) \rightarrow ((P \vee R) \rightarrow (Q \vee R)))$

T3.9. $(((P \rightarrow Q) \wedge (R \rightarrow S)) \rightarrow ((P \wedge R) \rightarrow (Q \wedge S)))$

T3.10. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \wedge Q) \rightarrow R))$

T3.10a. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.10b. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.11. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow (Q \rightarrow (P \rightarrow R)))$

T3.11a. $((P \rightarrow (Q \rightarrow R)) \rightarrow (Q \rightarrow (P \rightarrow R)))$

T3.11b. $((Q \rightarrow (P \rightarrow R)) \rightarrow (P \rightarrow (Q \rightarrow R)))$

T3.12. $((P \rightarrow Q) \leftrightarrow (\sim P \vee Q))$

T3.12a. $((P \rightarrow Q) \rightarrow (\sim P \vee Q))$

T3.12b. $((\sim P \vee Q) \rightarrow (P \rightarrow Q))$

T3.13. $((P \rightarrow Q) \leftrightarrow \sim (P \wedge \sim Q))$

T3.13a. $((P \rightarrow Q) \rightarrow \sim (P \wedge \sim Q))$

T3.13b. $(\sim (P \wedge \sim Q) \rightarrow (P \rightarrow Q))$

T3.14. $((P \rightarrow \sim P) \leftrightarrow \sim P)$

T3.14a. $((P \rightarrow \sim P) \rightarrow \sim P)$

T3.14b. $(\sim P \rightarrow (P \rightarrow \sim P))$

T3.15. $((P \rightarrow (Q \wedge \sim Q)) \leftrightarrow \sim P)$

T3.15a. $((P \rightarrow (Q \wedge \sim Q)) \rightarrow \sim P)$

T3.15b. $(\sim P \rightarrow (P \rightarrow (Q \wedge \sim Q)))$

T3.16. $(P \leftrightarrow P)$

T3.17. $\sim (P \leftrightarrow \sim P)$

T3.18. $(\sim (P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q))$

T3.18a. $(\sim (P \leftrightarrow Q) \rightarrow (P \leftrightarrow \sim Q))$

T3.18b. $((P \leftrightarrow \sim Q) \rightarrow \sim (P \leftrightarrow Q))$

T3.19. $((P \leftrightarrow Q) \leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)))$

T3.19a. $((P \leftrightarrow Q) \rightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)))$

T3.18b. $(((P \rightarrow Q) \wedge (Q \rightarrow P)) \rightarrow (P \leftrightarrow Q))$

T3.20. $((P \leftrightarrow Q) \leftrightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)))$

T3.20a. $((P \leftrightarrow Q) \rightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)))$

T3.20b. $(((P \wedge Q) \vee (\sim P \wedge \sim Q)) \rightarrow (P \leftrightarrow Q))$