3.14.1. Conditional Deduction Problems

A. Provide a **justification** for each line of the following conditional deductions.

(1)

1. $(P \vee Q)$
2. $(\sim P \vee R)$

Get $(\sim R \rightarrow Q)$

3. $\sim R$
4. $\sim P$
5. Q

6. $(\sim R \rightarrow Q)$

(2)

Get: $(\sim P \rightarrow ((P \vee Q) \rightarrow Q))$

1. $\sim P$

Get: $((P \vee Q) \rightarrow Q)$

2. $(P \vee Q)$
3. Q

4. $((P \vee Q) \rightarrow Q)$

5. $(\sim P \rightarrow ((P \vee Q) \rightarrow Q))$

B. Provide a **conditional deduction** for each of the following arguments.

1. $((P \vee Q) \vee R)$. $(\sim P \vee R)$ $\therefore$ $(\sim R \rightarrow Q)$

2. $((P \wedge Q) \vee R)$ $\therefore$ $(\sim P \rightarrow R)$

3. $(\sim(P \rightarrow Q) \vee R)$ $\therefore$ $(Q \rightarrow R)$

4. $\sim(\sim P \rightarrow Q)$ $\therefore$ $\sim(P \vee Q)$

5. $\sim(P \rightarrow Q)$ $\therefore$ $(P \wedge \sim Q)$

C. **Translate** each of the following arguments into the formal language (including your **translation key**). Then provide a **conditional deduction** for each argument.

1. Letitia didn't study for the exam. $\therefore$ If Letitia passed the exam, she did so without studying for it.

2. We're having truffles. $\therefore$ If we're not having both truffles and grog then we're not having grog.

3. We're not having both truffles and grog. $\therefore$ If we're having truffles then we're not having grog.

4. Barbie didn't go out without taking her umbrella. $\therefore$ If Barbie went out, she took her umbrella.

(*See* 2.10.1 § 3 *on translating a negated "*without*" sentence.*)

D. **Translate** each of the following sentence into the formal language (including your **translation key**). Then provide a **proof** for each argument.

1. If Letitia didn't study for the exam, then if she passed it she did so without studying for it.

2. If Neko's not going to Blazing Cat then assuming Jack's not going neither of them are.

3. Assuming we're having truffles, if we're not having both truffles and grog then we're not having grog.

E. It was noted in 3.6 that every argument in the formal language has a corresponding **leading principle**: a conditional whose antecedent is the conjunction of that argument's premises, and whose consequent is the conclusion of the argument. So the following argument has the leading principle listed below.

1. If Rex's team lost, then Rex is upset.	1. $(P \rightarrow Q)$
2. Rex's team lost.	2. P
∴ Rex is upset.	$\therefore Q$

Leading Principle: $(\ \underline{(P \rightarrow Q) \wedge\ P}\ ) \rightarrow Q\ )$

Armed now with conditional deduction, show that **whenever an argument is valid** (having a deduction of its conclusion from its premises), **there's a proof of the argument's leading principle**.