

3.16.1. Derived Rule Problems

For each of the following deductive systems, show that one or more of our deductive rules are **derived rules** in that system (by constructing a deduction of that rule, in that system).

1. Show that the rule **MP** can be treated as a derived rule in our Chapter Three deductive system, by providing a deduction of the argument “ $(P \rightarrow Q) \cdot P \therefore Q$ ” that doesn’t use MP.

2. Show that the rule **MT** can be treated as a derived rule in our Chapter Three deductive system, by providing a deduction of the argument “ $(P \rightarrow Q) \cdot \sim Q \therefore \sim P$ ” that doesn’t use MT.

3. The deductive system **DS1** has only ID, plus the the rules $\wedge-$, $\wedge+$, $\vee-$, $\vee+$, and “Negated Conditional” (“ $\sim \rightarrow$ ”).

Negated Conditional ($\sim \rightarrow$)

$$\frac{\sim(\bullet \rightarrow \blacktriangle)}{\therefore (\bullet \wedge \sim \blacktriangle)} \qquad \frac{(\bullet \wedge \sim \blacktriangle)}{\therefore \sim(\bullet \rightarrow \blacktriangle)}$$

Thanks to $\sim \rightarrow$, any **CD** from our system of deduction can be converted into an ID in DS1.¹

3a. Show that **MT** is a derived rule in DS1.

3b. Show that **MP** is a derived rule in DS1.

¹ Because DS2 has **ID** and the rules $\vee-$ and $\vee+$, the rules R, $\sim-$, and $\sim+$ are already derived rules in this system – as shown in 2.40.1 Problems 2, 3, and 5.

4. The deductive system **DS2** is like our Chapter Three deductive system except that it lacks the rule $\vee-$ and instead has the rule **Separation of Cases (SC)**.

Separation of Cases (SC)

$$\begin{array}{c}
 (\bullet \vee \heartsuit) \\
 (\bullet \rightarrow \blacktriangle) \\
 (\heartsuit \rightarrow \blacktriangle) \\
 \hline
 \blacktriangle
 \end{array}$$

So the following argument is an example of (SC).

$$\begin{array}{c}
 1. (P \vee Q) \\
 2. (P \rightarrow R) \\
 3. (Q \rightarrow R) \\
 \hline
 \therefore R
 \end{array}$$

4a. Show that the above argument is deducible in the Chapter Three deductive system (and hence that **SC** can act as a derived rule in the Chapter Three system).

4b. Show that the following argument is deducible in the system **DS3** (and hence that $\vee-$ is a derived rule in **DS3**) using only **CD**, **ID**, **R**, and **SC**.

$$\begin{array}{c}
 1. (P \vee Q) \\
 2. \sim P \\
 \hline
 \therefore Q
 \end{array}$$

5. The following argument is an instance of the rule **Double Disjunction** (DD), discussed in Chapter Two.

$$\begin{array}{l} 1. (P \vee Q) \\ 2. (P \vee \sim Q) \\ \hline \therefore P \end{array}$$

Show that Double Disjunction is a derived rule, by providing a deduction of the above argument using only $\sim+$, $\vee-$, **SC**, and **CD**.

6.

Separation of Cases (SC)

$$\begin{array}{l} (\bullet \vee \heartsuit) \\ (\bullet \rightarrow \blacktriangle) \\ (\heartsuit \rightarrow \blacktriangle) \\ \hline \blacktriangle \end{array}$$

Negated Conjunction ($\sim\wedge$)

$$\begin{array}{l} \sim \bullet \\ \hline \therefore \sim(\bullet \wedge \blacktriangle) \end{array}$$

Provide a deduction for the following argument using only **SC**, $\sim\wedge$, and **CD**.

$$\begin{array}{l} (\sim P \vee \sim Q) \\ \hline \therefore \sim(P \wedge Q) \end{array}$$

7. Note that the argument “ $(P \rightarrow (P \rightarrow Q)) \therefore (P \rightarrow Q)$ ” is valid. We can use the rule Separation of Cases to **explain** the validity of this rule **deductively**: from the sentence “ $(P \rightarrow (P \rightarrow Q))$,” when accompanied by the sentences “ $(P \vee \sim P)$ ” and “ $(\sim P \rightarrow (P \rightarrow Q))$,” the conclusion “ $(P \rightarrow Q)$ ” follows as an instance of Separation of Cases.

$$\begin{array}{l}
 1. (P \vee \sim P) \\
 2. (P \rightarrow (P \rightarrow Q)) \\
 3. (\sim P \rightarrow (P \rightarrow Q)) \\
 \hline
 \therefore (P \rightarrow Q) \quad 1, 2, 3, \text{SC}
 \end{array}$$

But the other two premises, “ $(P \vee \sim P)$ ” and “ $(\sim P \rightarrow (P \rightarrow Q))$,” are **theorems** provable without appeal to premises² – in effect, built in the deductive system. In that sense, from the deductive system alone “ $(P \rightarrow (P \rightarrow Q))$ ” entails “ $(P \rightarrow Q)$ ” by Separation of Cases.

For each of the following arguments, use **Separation of Cases** plus theorems to deductively explain the validity of that argument.

7a. $(P \vee Q) \cdot (P \rightarrow Q) \therefore Q$

7b. $(\sim P \rightarrow P) \therefore P$

7c. $(P \rightarrow \sim(P \wedge Q)) \therefore \sim(P \wedge Q)$

7d. $(P \rightarrow (Q \rightarrow \sim P)) \therefore (Q \rightarrow \sim P)$

7e. $((P \rightarrow Q) \rightarrow P) \therefore P$

² “ $(P \vee \sim P)$ ” is theorem **T2.2** from 2.41.1, and “ $(\sim P \rightarrow (P \rightarrow Q))$ ” is theorem **T3.3b** from 3.13.1 C.

