

1.8. Validity Counterexamples, And an Informal Test of Validity

1. Validity Counterexamples. Understanding logic as the study of **validity** – whether the conclusion of an argument **follows from** its premise(s) – we begin developing a test of validity. In its first form this test will remain informal and intuitive; but it will contain the elements essential to later, more sophisticated tests. Central to all these is a concept hinted at in previous examples of invalid arguments.

Recall that we judged Argument B invalid because it seems possible for the premises of B to be true while the conclusion is false.

Argument B

$$1+1=2$$

$$2+2=4$$

∴ The first U.S. president was born in Boston.

Such a case reveals the invalidity of an argument because a *valid* argument should be immune to precisely this possibility. With a valid argument true premises are accompanied by true conclusion *without exception*, so there's no *possible* way of having true premises without a true conclusion. If there is a possible way for Argument B to have true premises but false conclusion, then Argument B doesn't fit the definition of a "valid argument".

We call such a possible situation a **validity counterexample** (or "counterexample," for short).

A **validity counterexample** for an argument is a **possible situation** where the **premises** of the argument are all **true**, but the **conclusion** is **false**.

For Argument B, the actual world – where $1+1$ does indeed equal 2, and $2+2$ equals 4, but the first U.S. president was not born in Boston – served as a validity counterexample. But we saw that with some arguments we need to stretch our imaginations to find a validity counterexample. Argument D illustrates this.

Argument D

$$1+1=2$$

$$2+2=4$$

∴ The first U.S. president was born in Westmoreland County, Virginia.

In the actual world, the premises and conclusion of D are all true. But that isn't enough to make D valid – since it's still *possible* for the argument to have true premises with a false conclusion. With just a bit of imagination we described such a possibility: a situation where John Adams won the first U.S. election, while the mathematical facts remained the same. That situation would qualify as a validity counterexample for Argument D – establishing that the argument is *invalid*.

Just one validity counterexample is sufficient to prove an argument invalid. Indeed, that point was made clear by one of our alternate definitions of “valid argument” in the last section.

Valid argument: an argument where it's impossible to have true premises without having a true conclusion.

If we can show, for a given argument, that it *is* possible for it to have true premises without a true conclusion, we've shown that the argument falls short of being a valid argument.

For that reason the search for validity counterexamples takes center stage when testing an argument for validity. Roughly speaking: when testing an argument for validity we **try to think of a validity counterexample** for that argument. If we succeed in thinking up such a possibility, we know that the argument is invalid – and if invalid, unconvincing.

We won't rest content with this seat-of-the-pants, imagination-based test of validity. But already it provides an opportunity to rehearse the core concepts of **validity** and **validity counterexamples**, before grappling with more advanced tests. For even in those later tests, these two notions remain central.

2. Validity Counterexample Examples. The following simple argument supplies a bit of practice in picking out validity counterexamples.

1. All of the birds I've seen are small and yellow.

∴ All birds are small and yellow.

We walk through a series of possibilities, asking for each whether it qualifies as a validity counterexample for this argument. First, consider Situation A.

Situation A: I've seen 20 small yellow birds; and there are hundreds of small birds, of various colors, that I haven't seen.

In Situation A the premise of our argument would be true. But the conclusion of the argument – that “All birds are small and yellow” – is certainly false in A. Since Situation A makes all the premises of the argument true (all *one* of them) while making the conclusion false, Situation A qualifies as a **validity counterexample** for the argument.

Argument	Situation A
1. All of the birds I've seen are small and yellow.	TRUE
∴ All birds are small and yellow.	FALSE

Thanks to Situation A, we know this argument is **invalid**: true premises do *not* guarantee a true conclusion here, so the conclusion does *not* follow from the premise. (Equivalently: the premises don't **entail** the conclusion.)

For a bit more practice, we continue with Possible Situation B.

Situation B: I've seen 20 birds; and all the birds, whether I've seen them or not, are small and yellow.

In B the premise of the argument is **true**: if all the birds are small and yellow, then certainly all the ones I've seen are. And of course the conclusion is **true** here as well.

Argument	Situation B
<u>1. All of the birds I've seen are small and yellow.</u>	TRUE
∴ All birds are small and yellow.	TRUE

Situation B is **not** a validity counterexample for this argument. This situation tells us **nothing** about the validity of the argument.

How about Possible Situation C?

Situation C: All the birds are big and white; and I've seen 20 of them.

In such a situation the premise of the argument would be **false**: here it is not true that all the birds I've seen are small and yellow. The conclusion would be **false** in C as well.

Argument	Situation C
<u>1. All of the birds I've seen are small and yellow.</u>	FALSE
∴ All birds are small and yellow.	FALSE

Situation C does **not** qualify as a validity counterexample for this argument. C tells us **nothing** about this argument's validity.

We said that Situations B and C tell us nothing about the validity of the argument. Here's why.

Situation A already established that this argument is invalid. But Situation B shows us something important about an invalid argument: **an invalid argument can** (by lucky coincidence) **have true premises and true conclusion**.

Argument	Situation B
1. All of the birds I've seen are small and yellow.	TRUE
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∴ All birds are small and yellow.	TRUE

Of course a **valid** argument can also have true premises and conclusion – as this example shows.

Valid Argument:

1. George Washington was the first US president.
 2. George Washington was born in Virginia
-
- ∴ The first US president was born in Virginia.

In the actual world, for instance, the premises and conclusion of this valid argument are true.

Having true premises and true conclusion is something both valid and invalid arguments can do. So a situation like that – where the argument's premises and conclusion are all true – is no help in settling whether the argument is valid or invalid.

In Situation C the above invalid argument had false premises and a false conclusion.

Argument	Situation C
1. All of the birds I've seen are small and yellow.	FALSE
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∴ All birds are small and yellow.	FALSE

But that can happen to a valid argument as well.

Valid Argument:

1. Benjamin Franklin was the first US president.
 2. Benjamin Franklin was born in Florida
-
- ∴ The first US president was born in Florida.

This argument is **valid**, because if the premises **were** both true, the conclusion would have to be true as well. Of course, in the actual world both the premises and conclusion are **false**. So: a valid argument can have false premises and false conclusion (in a given situation).

Here again, **having false premises and false conclusion** (in a given situation) **is something both valid and invalid arguments can do**. So finding such a situation – where an argument's premises and conclusion are both false – tells us nothing about whether that argument is valid.

As single situations go, the **only** case that tells us anything about an argument's validity is a situation where the argument has true premises and a false conclusion – a **validity counterexample**. For that's the one sort of situation a valid argument will **never** find itself in.

So if, by scouring the world before us or any other possibilities the imagination can dream up, we find such a situation for a given argument, we know for certain that the argument is invalid. That's why the search for validity counterexamples plays such a central role in tests of validity.

Note finally that, just as we think of validity as a more technical version of plain old ‘following from,’ validity counterexamples are likewise something we refer to naturally in conversation (if not by that name). So when we want to object that the conclusion of an argument doesn’t follow from its premises, we say: “even if all that’s true, it doesn’t follow that....” In saying this we’re depicting a situation where all the premises are true yet the conclusion isn’t. And that’s just a validity counterexample for the argument in question. Validity counterexamples are what we naturally reach for to show that the conclusion **doesn’t follow** from the premises.

Summary: Validity Counterexamples

- A **validity counterexample** for an argument is a **possible situation where** the argument has **true premises** but a **false conclusion**.
- Finding a validity counterexample for an argument establishes that the argument is **invalid**. (As single situations go, no other kind of possible situation tells us *anything* about whether an argument is valid or invalid.)