2.18. Truth Tables and Validity

Of course our main semantic interest has been in the **validity** of arguments. And formal semantics provides a simple procedure for assessing validity.

By definition, a **valid argument** is an argument such that: *in every possible situation where the premises are true, the conclusion is true*. With valuations as formal stand-ins for possible situations, we sum up validity for formal arguments like so.[1]

> A formal argument is **valid** if (and only if): **every valuation making all the premises true** also **makes the conclusion true**.

The following English argument was one of our earliest and clearest examples of a valid argument.

1. Either the Chess Club won the prize, or the Surf Club won the prize.	1. $(P \vee Q)$
2. The Chess Club didn't win the prize.	2. $\sim P$
∴ 3. The Surf Club won the prize.	∴ 3. Q

To test this English argument for validity using truth tables, we translate it into formal language (already done here), then construct a truth table for each premise and the conclusion.

[1] We could, if we liked, state this point in the jargon of our formal semantics: a formal argument is valid if (and only if) each valuation which **simultaneously satisfies** the premises also **satisfies** the conclusion

Those truth tables look like this.[2]

$$(P \vee Q) \;.\; {\sim}P \;\therefore\; Q$$

		(1)	(2)	∴
P	Q	(P ∨ Q)	~P	Q
1	1	1	0	1
1	0	1	0	0
⇨ 0	1	**1**	**1**	**1**
0	0	0	1	0

If this argument is valid, each valuation **making all the premises true** will make the **conclusion true** as well. Now, only one valuation here makes *all* the premises true: the third valuation (marked with an arrow). And that valuation does indeed make the conclusion true as well.

Here every case of true premises is a case of true conclusion – making this argument **valid**. Once again the formal semantics agrees with our intuitions about clear, simple cases – here, judgments of validity (or "following from").

By contrast, the next argument seems intuitively **invalid**.

Either we're having ice cream
or we're having cake.

We're having ice cream.

∴ We're having cake.

(R ∨ S)

R

∴ S

[2] Here, and in the examples that follow, I put a second copy of the conclusion at the right end. This isn't strictly necessary, of course – the truth table for conclusion "Q" already appeared earlier. But it is more natural to read the truth tables **from left to right**: from premises "(P ∨ Q)" and "~P" to conclusion "Q," so for convenience I make a second copy of "Q" on the right.

Two valuations make all the premises true: the first and second.

(2)		**(1)**	∴
R	S	**(R ∨ S)**	S
1	1	**1**	1
1	0	**1**	0
0	1	1	1
0	0	0	0

But not all those valuations make the conclusion true: the second valuation makes the conclusion **false**.

(2)		**(1)**	∴
R	S	**(R ∨ S)**	S
1	1	1	1
1	0	**1**	**0**
0	1	1	1
0	0	0	0

Here true premises are **not** always accompanied by a true conclusion. The formal semantics agrees with us that this argument is **invalid**.

And notice what that second valuation amounts to: making all the premises true but the conclusion false, it is a formal version of a **validity counterexample**.

Everything proceeds here just as in informal logic: even one validity counterexample renders the argument invalid; and a valid argument is one with *no* validity counterexamples.

So we can approach a formal test of validity in two different ways.

- **Method 1**: Check whether every valuation making (all of) the premises true also makes the conclusion true.
- **Method 2**: Sweep the valuations in search of a validity counterexample.

But at bottom these are just two ways of viewing the same procedure. For to be a validity counterexample, a valuation must (a) make the premises true, yet (b) make the conclusion false. To search for a valuation that does both (following Method 2), we first pick out the valuations meeting Condition (a) – the valuations **making all the premises true**; and then, to check for Condition (b), see whether the conclusion is true or false in each of those selected valuations.

But that’s just what we do when following Method 1: (a) pick out the valuations **making all the premises true**, then (b) check whether the conclusion is true or false in those selected valuations.

The two methods end up being equivalent.

So in practice our truth table test of validity works as follows.

Truth Table Test of Validity:

1. Pick out those valuations making *all* the premises true.
2. Check the conclusion in those selected valuations:
  a. If the conclusion is true in *every one* of those valuations, then the argument is **valid**.
  b. If the conclusion is false in *even one* of those selected valuations, then the argument is **invalid**.

Summary: Truth Tables and Validity

Truth Table Test of Validity:

0. **Build a truth table for each premise and** for the **conclusion**. (Build them all out of the same initial sentence letters, as one long truth table.)

1. Pick out those valuations making **all the premises true**.
2. Check the conclusion in those selected valuations:
- **a**. If the **conclusion is true in every one** of those valuations, the argument is **valid**.
- **b**. If the **conclusion is false in even one** of those selected valuations, then the argument is **invalid**.