

### 5.5.1. Quantifier Semantics Problems

**A. Translate** each of the following English sentences into the formal language, and then decide if that sentence is **true or false** in the model given below.

**Translation Key:**

**A:** Jack

**G:** \_\_ is impressive

**B:** The Cathedral of Learning

**H:** \_\_ is made of limestone

**Sentences:**

1. If Jack is impressive, then something is impressive.
2. If the Cathedral of Learning isn't impressive, then nothing is.
3. Not everything is made of limestone, but the Cathedral of Learning is.
4. Something is impressive if and only if not everything is unimpressive.

**Model:**

**D: { Jack, The Cathedral of Learning }**

**A: Jack**

**B: The Cathedral of Learning**

**G: { The Cathedral of Learning }**

**H: { The Cathedral of Learning }**

**B. Translate** each of the following English arguments into the formal language, and then decide if the model given below is a **validity counterexample** for that argument.

**Translation Key:**

**A:** Neko

**B:** Rex

**G:** \_\_ is made of matter

**H:** \_\_ is a building

**I:** \_\_ is a cat

**Arguments:**

(1) 1. Neko isn't a building made of matter.

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∴ Not everything is made of matter.

(2) 1. If Rex is a cat then something is a cat, and if Neko is a cat  
then something is a cat.

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∴ If either Rex or Neko is a cat, then something is a cat.

**Model:**

**D:** { **Neko**, **Rex** }

**A:** **Neko**

**B:** **Rex**

**G:** { **Neko**, **Rex** }

**H:** { }

**I:** { **Neko** }

C. Explain why the universal sentence “ $\forall x Gx$ ” will entail each of its instances – that is, why each of the following arguments will be valid.

$$\begin{array}{ccc} \frac{\forall x Gx}{\therefore GA} & \frac{\forall x Gx}{\therefore GB} & \frac{\forall x Gx}{\therefore GC} \quad (\text{etc.}) \end{array}$$

*(If one of these arguments were invalid, what would the validity counterexample for that argument look like?)*

D. Intuitively the following sentence looks **inconsistent**.

Everything is made of matter, but nothing is made of matter.

We translate this sentence like so.

**G:** \_\_ is made of matter

$(\forall x Gx \wedge \sim \exists x Gx)$

In 5.3 §2 we laid down a requirement on models that the domain of discourse not be empty. **Using a model with at least one object** in its domain, does the formal semantics find this sentence **inconsistent**?

If the domain is **empty** (containing no objects), does the formal semantics find this sentence **inconsistent**? (Does the extension of “G” in this model contain every object in the domain?)

**E.** Recall that we defined the Tilde Insertion Dual of a sentence as follows.

The **Tilde Insertion Dual** of a sentence is the result of placing a tilde before each sentence letter in the sentence, and before the entire sentence.

Using Tilde Insertion Duality, along with Quantifier Negation, to determine the dual of each of the following sentences.

$\forall x Gx$

$\exists x Gx$