2.17.1. Semantic Problems:
Tautology, Contradiction, and Logical Equivalence

A. **Translate** each English sentence into the formal language and build a **truth table** for that formal sentence. On the basis of that truth table, find a **simpler formal sentence** that's **logically equivalent** to the original; then **translate** that simpler sentence back **into English**.

1. Neko will either eat fish or eat fish.

2. Letitia's not a goth, and she's also not a goth.

3. Jack is a cat who either scales cliffs or doesn't.

4. Either Jack is a cat who scales cliffs, or he's one who doesn't.

5. Either Jake will attend Logicpalooza, or both he and Jezebel will.

6. Either Jake will attend Logicpalooza, or either he or Jezebel will.

7. Barbie went out, and she did so without taking her umbrella.

8. Elvis is a gambler, but one who doesn't drink whiskey.

9. Gasoline fights are either dangerous or pleasant, and they're also either dangerous or unpleasant.

10. Either Suki is neither tall nor tall and skinny, or she's neither tall nor skinny.

11. Either Trixie finished her homework, or she went to the party without finishing it.
(*Note: the simpler sentence doesn’t appear earlier in the truth table.*)

12. Lucretia cooked breakfast, but she didn’t do so without making a mess.
(*Hint*: *see the discussion of negated “without” sentences in* 2.10 §3.
Note: the simpler sentence doesn’t appear earlier in the truth table.)

B. For each pair of sentences, **translate** both into formal language and build **truth tables** for them to show that the two sentences are **logically equivalent**.

1a. Neither Trixie nor Elvis failed to attend the poker tournament.
1b. Both Trixie and Elvis attended the poker tournament.

2a. Letitia and Lucretia didn’t both fail to attend goth night at Novo.
2b. Either Letitia or Lucretia attended goth night at Novo.

3a. We’re having truffles, and either grog or grappa.
3b. We’re having either truffles and grog or truffles and grappa.

4a. Either we’re having truffles or we’re having grog and grappa.
4b. We’re having either truffles or grog, and either truffles or grappa.

5a. Neko is a cat who likes fish, and one who also likes cream cheese.
5b. Neko is a cat who likes both fish and cream cheese.

6a. Kitty’s not a singer who won a voice scholarship.
6b. Kitty’s not a singer unless she’s one who didn’t win a voice scholarship.
(*Hint*: *see the discussion of negations containing a relative clause in* 2.10 §3.)

7a. We won’t have a day off unless there’s a good reason.
7b. We won’t have a day off without there being a good reason.
(*Hint*: *see the discussion of negated “without” sentences in* 2.10 §3.)

C. For each numbered group of sentences below, use truth tables to show that all the sentences in that group are **logically equivalent**.

Distribution

1a. $(P \wedge (Q \vee R)$

1b. $((P \wedge Q) \vee (P \wedge R))$

2a. $(P \vee (Q \wedge R))$

2b. $((P \vee Q) \wedge (P \vee R))$

Idempotence

3a. $(P \vee P)$

3b. $(P \wedge P)$

3c. P

Absorption

4a. $(P \vee (P \wedge Q))$

4b. $(P \wedge (P \vee Q))$

4c. P

D. **Translate** each of the following sentences into formal language; then use a **truth table** to decide whether that sentence is a **tautology**, a **contradiction**, or **neither**.

1. Dr. Slim is a weasel who isn’t a weasel.
2. Jack either scales cliffs or doesn’t.
3. Jack is a cat who either scales cliffs or doesn’t.
4. Unless we’re having both truffles and grog, we’re not having truffles.
5. Unless we’re not having truffles, we’re having either truffles or grog.
6. Suki passed Business Logic without studying for the Business Logic exam, though she did study for the Business Logic exam.
7. Unless Rex’s handwriting is illegible, it’s both legible and beautiful.
8. Unless Rex’s handwriting is illegible, it’s either legible or beautiful.
9. Although Rex’s handwriting is illegible, it’s both legible and beautiful.
10. Unless Dr. Slim is a physician, he’s not a physician who performs surgery.
11. Unless Dr. Slim is a physician who performs surgery, he’s not a physician.
12. Dick and Dora both ordered a Sloe Gin Fizz, unless neither of them did.
13. Either Dick ordered a Sloe Gin Fizz or Dora ordered one – unless neither of them did.

E. Suppose we say that an **instance of a sentence** S is the result of **substituting** some formal sentence for a sentence letter in Sentence S. So (2) and (3), for example, are each an instance of Sentence (1).

(1) (**P** ∨ ~**P**)
(2) (**Q** ∨ ~**Q**)
(3) (**(R ∧ ~S)** ∨ **~(R ∧ ~S)**)

If a sentence is a tautology – as (1) is – we expect **any instance of that sentence** to also be a **tautology** (as both (2) and (3) are). But if we substitute a sentence **only** for **the first occurrence** of "P" in (1) we get Sentence (4), which isn't a tautology.

(4) (**Q** ∨ ~**P**)

And if we replace every "P" in (1), but **substitute different sentences** for the different occurrences of "P," we get Sentence (5), which is also not a tautology.[1]

(5) (**Q** ∨ ~**R**)

Likewise while (6) is a tautology, if we substitute "R" for all occurrences of "P" and for the first occurrence of "Q" the result is the non-tautology (7).

(6) ((**P** ∨ ~**P**) ∧ (Q ∨ ~Q))
(7) ((**R** ∨ ~**R**) ∧ (**R** ∨ ~Q))

State conditions for being an instance of a sentence which give the right results – namely, that **any instance of a tautology is a tautology**.[2]

In particular: what **conditions** needs to be added on **how we substitute a sentence for a sentence letter**, in order to block (4) and (5) from counting as instances of (1), and to block (7) from counting as an instance of (6)?[3]

F. According to the definition of "instance" from **(E)**, is it also true that **every instance of a contradiction is a contradiction**?

---

[1] In fact if we substitute "(P ∧ ~P)" for the left "P" in (1) and "(P ∨ ~P)" for the right "P" we get a contradiction: "((P ∧ ~P) ∨ ~(P ∨ ~P))".

[2] These conditions on being an instance parallel those for being an **instance of a formula** discussed in 5.8.

[3] Such substitution is sometimes called "uniform substitution".

G. Will every **instance of a consistent sentence** be a consistent sentence?

H. **Consistent and Inconsistent Sentences**. Every inconsistent set of sentences we've looked at so far has contained either **(a)** a sentence that's a **contradiction**, such as {(P ∧ ~P)}; or **(b)** a pair of **contradictory sentences** – sentences where one is the negation of the other, such as {P, ~P}.

Build an inconsistent set of sentences which has **neither** feature **(a)** nor feature **(b)**.

I. **A Puzzle about Disjunctions and Conjunctions.** We've been counting "unless" as a disjunction phrase, translated by the vel.

> **P**: Rex will go
> **Q**: Barbie (will) go

> 1. Rex won't go **unless** Barbie goes. (~P ∨ Q)

But note that "unless" can appear accompanied by the words "too" or "as well".

> 2. Rex won't go **unless** Barbie goes **too**.
> 3. Rex won't go **unless** Barbie goes **as well**.

We noted earlier that "too" and "as well" typically accompany conjunction phrases.

> 4. The movie is entertaining, **and** it's informative **too**.
> 5. Kids will enjoy the movie, **but** adults will like it **as well**.

That suggests that Sentences (2) and (3) should instead be translated with a **conjunction as its right part** (meaning "Rex won't go unless both he and Barbie go").

> Rex won't go **unless** Rex goes and Barbie goes **too**. (~P ∨ <u>(P ∧ Q)</u>)
> Rex won't go **unless** Rex goes and Barbie goes **as well**. (~P ∨ <u>(P ∧ Q)</u>)

Build **truth tables** for "(~P ∨ Q)" and "(~P ∨ (P ∧ Q))" to show why **there's no semantic reason to make this change** – so that, for purposes of truth and validity, we can continue translating sentences (2) and (3) as simply "(~P ∨ Q)".

J. Build a truth table for each of the following sentences, to show that the sentence is a **tautology**.

T 2.1. $\sim(P \wedge \sim P)$

T 2.2. $(P \vee \sim P)$

T 2.3. $(P \vee \sim(P \vee P))$

T 2.4. $(P \vee (\sim P \vee \sim P))$

T 2.5. $(P \vee (\sim P \vee Q))$

T 2.6. $(\sim Q \vee (\sim P \vee Q))$

T 2.7. $((P \vee Q) \vee \sim(P \wedge Q))$

T 2.8. $((P \vee Q) \vee (\sim P \wedge \sim Q))$

T 2.9. $((\sim(P \vee Q) \vee \sim(\sim P \vee Q)) \vee Q)$

T 2.10. $(((P \wedge Q) \vee (\sim P \wedge Q)) \vee \sim Q)$

T 2.11. $(((P \vee Q) \wedge (\sim P \vee Q)) \vee \sim Q)$

T 2.12. $\sim(((P \vee Q) \wedge (\sim P \vee Q)) \wedge \sim Q)$