3.16. Alternative Systems of Rules and Deduction

The idea of a derived rule of inference is already familiar from Chapter Two: a derived rule of inference is a convenient rule which is not one of the basic rules of a deductive system, but for which a deduction can be provided using only the basic rules of that system. De Morgan's Law was adopted as a derived rule in the Chapter Two deductive system. With the addition of further basic rules of inference in the Chapter Three deductive system, new possibilities for derived rules arise as well.

In particular: we will here explore alternative sets of rules (all derived rules in our deductive system) whose adoption as basic rules would allow us to treat current parts of our system as derived.

1. Eliminating Conditional Deduction. We also added a new form of deduction – Conditional Deduction (CD) – devoted only to deducing conditionals. But with the introduction of one or another further rule, we could always

Add a new rule, **Non-Contradiction** (**NC**):

$$\frac{\qquad\qquad}{\therefore \sim(\bullet \wedge \sim\bullet)}$$

Then replace each ID with a CD proving a sentence of this form.[1]

If (AID), then $(\bullet \wedge \sim\bullet)$

[1] Note that Suppes 1957: xx treats ID as a form of CD in just this way. Recall also that the conditional derived via CD, with a contradiction as consequent, basically repeats the sentence used to get the negation truth table in the language $\{\rightarrow, \perp\}$: "$(P \rightarrow \perp)$" is logically equivalent to "$\sim P$".

Eliminating ID: An Example

Old Way:

1.	$\sim P$	
		Get: $\sim(P \wedge Q)$ (ID)
2.	$\sim\sim(P \wedge Q)$	AID
3.	$(P \wedge Q)$	1, $\sim -$
4.	P	3, $\wedge -$
5.	$\sim P$	1, R
6.	$\sim(P \wedge Q)$	2, 4, 5, CD

New Way:

1.	$\sim P$	
		Get: $(\sim\sim(P \wedge Q) \rightarrow (P \wedge \sim P))$ (CD)
2.	$\sim\sim(P \wedge Q)$	ACD
3.	$(P \wedge Q)$	2, $\sim -$
4.	P	3, $\wedge -$
5.	$\sim P$	1, R
6.	$(P \wedge \sim P)$	4, 5, $\wedge +$
7.	$(\sim\sim(P \wedge Q) \rightarrow (P \wedge \sim P))$	2, 6, CD
8.	$\sim(P \wedge \sim P)$	NC
9.	$\sim\sim\sim(P \wedge Q)$	7, 8, MT
10.	$\sim(P \wedge Q)$	9, $\sim -$

2. Eliminating CD (I). We also added a new form of deduction – Conditional Deduction (CD) – devoted only to deducing conditionals. But with the introduction of one or another further rule, we could always

Add a new rule, **Negated Conditional** (~→):

$$\frac{\sim(\bullet \rightarrow \blacktriangle)}{\therefore\ (\bullet \wedge \sim\blacktriangle)}$$

Then deduce conditionals by using ID.

(*Note that the converse rule*

$$\frac{(\bullet \wedge \sim\blacktriangle)}{\therefore\ \sim(\bullet \rightarrow \blacktriangle)}$$

is already deducible using ID and MP.)

Eliminating CD: An Example

Old Way:

1.	$(P \to Q)$	
2.	$(Q \to R)$	
		Get: $(P \to R)$ (CD)
3.	P	ACD
4.	Q	1, 3, MP
5.	R	2, 4, MP
6.	$(P \to Q)$	3, 5, CD

New Way:

1.	$(P \to Q)$	
2.	$(Q \to R)$	
		Get: $(P \to R)$ (ID)
3.	$\sim(P \to R)$	AID
4.	$(P \wedge \sim R)$	3, $\sim\to$
5.	P	4, $\wedge-$
6.	Q	1, 5, MP
7.	R	2, 6, MP
8.	$\sim R$	4, $\wedge-$
9.	$(P \to Q)$	3, 7, 8, ID

3. Eliminating CD (II). Add a new rule, **Negation Disjunctiuon** (~∨).

$$\frac{(\sim\bullet \vee \blacktriangle)}{\therefore\ (\bullet \rightarrow \blacktriangle)}$$

Then deduce a conditional by first getting its disjunctive counterpart (here, the premise) through ID.

(*Note that the converse rule*

$$\frac{(\bullet \rightarrow \blacktriangle)}{\therefore\ (\sim\bullet \vee \blacktriangle)}$$

is already deducible using ID, DM, and MP.)