3.15.1. Deductions and Proofs: Problems

A. For each of the following formal arguments, show that the argument is valid by constructing a **deduction** of the argument.

1. $((P \lor Q) \to R)$ $\therefore$ $((P \lor Q) \to R)$

2. $((P \to Q) \to P)$ $\therefore$ $((P \to Q) \to Q)$

3. $(P \to R)$ $\therefore$ $((P \land Q) \to R)$

4. $(P \to Q)$. $(Q \to R)$ $\therefore$ $(P \to R)$

5. $((P \to Q) \to (R \to S))$. $(R \to T)$. $((R \to T) \to (P \to Q))$.
$(Q \to (S \to U))$. $(T \to P)$ $\therefore$ $(R \to U)$

6. $(P \to \sim Q)$. $(R \to Q)$. $(\sim R \to \sim S)$. $(T \to S)$ $\therefore$ $(P \to \sim T)$

7. $(P \to R)$. $(Q \to R)$ $\therefore$ $((P \lor Q) \to R)$

8. $(R \to (P \lor Q))$. $(Q \to P)$ $\therefore$ $(\sim P \to \sim R)$

9. $((P \lor Q) \to R)$. $(R \to S)$. $(T \to Q)$. $\sim S$ $\therefore$ $\sim T$

10. $(R \to (P \land Q))$ $\therefore$ $(\sim P \to \sim R)$

11. $(R \to (P \lor Q))$. $(P \to S)$. $((S \lor T) \to Q)$. $\sim Q$ $\therefore$ $\sim(R \lor T)$

12. $((P \land Q) \to R)$. $(\sim S \to (Q \land \sim R))$ $\therefore$ $(P \to S)$

B. **Translate** each of the following English arguments into the formal language of Chapter Three, then show that the argument is valid by constructing a **deduction** of it.

1. Either Dick or Dora is having a Gibson. If either of them is having a Gibson, Dick is. Dick is having a Gibson only if Dora is. ∴ Both Dick and Dora are having a Gibson.

2. Assuming Trixie passed Logic if the test wasn't too hard, Barbie passed Logic. If Trixie didn't pass Logic, then the test was too hard. ∴ Barbie passed Logic.

3. If Letitia's going to the party then Lucretia's not going. Letitia's going to the party if and only if Lucretia is. ∴ Neither Letitia nor Lucretia are going to the party.

4. If Kitty's getting a manicure, then she'll have a massage only if the check cleared. The check didn't clear, but Kitty's getting a manicure. ∴ Kitty won't have a massage.

(Can be done without ID if DM is used.)

5. Kitty will have both a manicure and a massage if the check cleared, and she'll have a manicure without (having) a massage otherwise. ∴ Kitty will have a manicure, and she'll have a massage if and only if the check cleared.

6. Jack's making a tuna sandwich if Neko's working on her invention, and a seafood casserole otherwise. Neko's working on her invention only if Jack's making a seafood casserole. ∴ Jack's making a seafood casserole.

7. Rex is making a tuna sandwich if Neko's working on her invention, and a seafood casserole otherwise. Neko's working on her invention if and only if Rex is making a seafood casserole. ∴ Neko's working on her invention and Rex is making a seafood casserole.

8. If the chef is the killer then Nick will catch him in a lie, assuming Nora joins the conversation. Provided that Nick will catch the chef in a lie if the chef is the killer, the chef will confess to the crime. The chef will confess to the crime only if he's the killer. ∴ If Nora joins the conversation Nick will catch the chef in a lie.

9. That consonantal segment is prevocalic if it occurs initially; otherwise it's voiceless. Provided it's either prevocalic or voiceless, it's both continuant and strident. Assuming it's continuant, it's tense if it's strident. If it's tense, then if it occurs initially it's palatalized. ∴ That consonantal segment is palatalized and voiceless.

(*Adapted from* Partee, ter Meulen and Wall 1990: 134, Problem 10e)

10. The president will sign an executive order if the bill stalls in either the House or the Senate. The Widget lobby will mobilize only if the bill stalls in the Senate. Assuming Gizmo PAC holds a phone campaign, the bill will stall in the House. If Gizmo PAC doesn't hold a phone campaign, the Widget lobby will mobilize. Therefore, the president will sign an executive order.

11. If neither the butler nor the chauffeur killed the baron, then the cook did. The cook killed the baron if and only if the stew was poisoned. The chauffeur killed the baron just in case there was a bomb in the car. The stew wasn't poisoned, and the butler didn't kill the baron. Therefore, there was a bomb in the car.

(*Adapted from* Partee, ter Meulen and Wall 1990: 134, Problem 10a)

12. Either Neko is a cat who can't stop eating, or Jack is a cat who's been stealing Neko's food. Neko can stop eating if Jack hasn't been stealing her food. Neko is a cat if and only if Jack is. Therefore, Jack is a cat who's been stealing Neko's food.

13. If God exists, then He is omnipotent. If God exists, then He is omniscient. If God exists, then He is benevolent. If God can prevent evil, then if He knows that evil exists, then He is not benevolent if He does not prevent it. If God is omnipotent, then He can prevent evil. If God is omniscient, then He knows that evil exists if it does indeed exist. Evil does not exist if God prevents it. Evil exists. Therefore, God does not exist.

(*from* Kalish, Montague, and Mar 1980: 35, Problem 35)

C. Show that each of the following sentences is a **theorem**, by constructing a **proof** of that sentence.

T3.1. $(P \to P)$

T3.2. $((P \to Q) \to ((Q \to R) \to (P \to R)))$

T3.3a. $(P \to (\sim P \to Q))$
T3.3b. $(\sim P \to (P \to Q))$

T3.4. $(P \to ((P \to Q) \to Q))$

T 3.5. $(((P \to Q) \to P) \leftrightarrow P)$
T 3.5a. $(((P \to Q) \to P) \to P)$
T 3.5b. $(P \to ((P \to Q) \to P))$

T 3.6. $(((P \to Q) \leftrightarrow (P \to (P \wedge Q))$
T 3.6a. $((P \to Q) \to (P \to (P \wedge Q)))$
T 3.6b. $((P \to (P \wedge Q)) \to (P \to Q))$

T3.7. $((P \to Q) \vee (Q \to P))$

T3.8. $((P \to Q) \to ((P \vee R) \to (Q \vee R)))$

T9. $(((P \to Q) \wedge (P \to R)) \leftrightarrow (P \to (Q \wedge R)))$
T9a. $(((P \to Q) \wedge (P \to R)) \to (P \to (Q \wedge R)))$
T9b. $((P \to (Q \wedge R)) \to ((P \to Q) \wedge (P \to R)))$

T3.10a. $(\sim P \to \sim(P \wedge Q))$
T3.10b. $(\sim Q \to \sim(P \wedge Q))$

T3.11. $(((P \to Q) \wedge (R \to S)) \to ((P \wedge R) \to (Q \wedge S)))$

T3.12. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow ((P \wedge Q) \rightarrow R))$

T3.12a. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.12b. $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R))$

T3.13. $((P \rightarrow (Q \rightarrow R)) \leftrightarrow (Q \rightarrow (P \rightarrow R)))$

T3.13a. $((P \rightarrow (Q \rightarrow R)) \rightarrow (Q \rightarrow (P \rightarrow R)))$

T3.13b. $((Q \rightarrow (P \rightarrow R)) \rightarrow (P \rightarrow (Q \rightarrow R)))$

T3.14. $((P \rightarrow \sim P) \leftrightarrow \sim P)$

T3.14a. $((P \rightarrow \sim P) \rightarrow \sim P)$

T3.14b. $(\sim P \rightarrow (P \rightarrow \sim P))$

T3.15. $((P \rightarrow (Q \wedge \sim Q)) \leftrightarrow \sim P)$

T3.15a. $((P \rightarrow (Q \wedge \sim Q)) \rightarrow \sim P)$

T3.15b. $(\sim P \rightarrow (P \rightarrow (Q \wedge \sim Q)))$

T3.16. $((P \rightarrow Q) \leftrightarrow (\sim P \vee Q))$

T3.16a. $((P \rightarrow Q) \rightarrow (\sim P \vee Q))$

T3.16b. $((\sim P \vee Q) \rightarrow (P \rightarrow Q))$

T3.17. $((P \rightarrow Q) \leftrightarrow \sim(P \wedge \sim Q))$

T3.17a. $((P \rightarrow Q) \rightarrow \sim(P \wedge \sim Q))$

T3.17b. $(\sim(P \wedge \sim Q) \rightarrow (P \rightarrow Q))$

T3.18. $((P \wedge P) \leftrightarrow P)$

T3.18a. $((P \wedge P) \rightarrow P)$

T3.18b. $(P \rightarrow (P \wedge Q))$

T3.19. $((P \vee P) \leftrightarrow P)$

T3.19a. $((P \vee P) \rightarrow P)$

T3.19b. $(P \rightarrow (P \vee Q))$

T3.20. $(P \leftrightarrow P)$

T3.21a. $\sim(P \leftrightarrow \sim P)$

T3.21b. $\sim(\sim P \leftrightarrow P)$

T3.22. $(\sim(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow \sim Q))$

T3.22a. $(\sim(P \leftrightarrow Q) \rightarrow (P \leftrightarrow \sim Q))$

T3.22b. $((P \leftrightarrow \sim Q) \rightarrow \sim(P \leftrightarrow Q))$

T3.23. $((P \leftrightarrow Q) \leftrightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)))$

T3.23a. $((P \leftrightarrow Q) \rightarrow ((P \rightarrow Q) \wedge (Q \rightarrow P)))$

T3.23b. $(((P \rightarrow Q) \wedge (Q \rightarrow P)) \rightarrow (P \leftrightarrow Q))$

T3.24. $((P \leftrightarrow Q) \leftrightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)))$

T3.24a. $((P \leftrightarrow Q) \rightarrow ((P \wedge Q) \vee (\sim P \wedge \sim Q)))$

T3.24b. $(((P \wedge Q) \vee (\sim P \wedge \sim Q)) \rightarrow (P \leftrightarrow Q))$

E. It was noted in 3.6 that every argument in the formal language has a corresponding **leading principle**: a conditional whose antecedent is the conjunction of that argument's premises, and whose consequent is the conclusion of the argument. So the following argument has the leading principle listed below.

1. If Rex's team lost, then Rex is upset.	1. $(P \rightarrow Q)$
2. Rex's team lost.	2. P
∴ Rex is upset.	$\therefore Q$

Leading Principle: $(\ \underline{(P \rightarrow Q) \wedge\ P}\ ) \rightarrow Q\ )$

Armed now with conditional deduction and Modus Ponens, show that **if an argument's leading principle is a theorem** (capable of a proof appealing to no premises), then the **argument is valid** (so: there's a deduction of that argument's conclusion from its premises).