2.40.1. Derived Rule Problems

1. Provide a deduction for the following argument, using only $\sim$**–**, and $\sim$**+**.

$$\begin{array}{l} 1.\ \ P \\ \hline \therefore\ P \end{array}$$

2. Provide a deduction for the argument in Problem 1 using only $\vee$+, $\vee$**–**, and **ID**.

3. Provide a deduction for the following argument, using only $\vee$+, $\vee$**–**, and **ID**.

$$\begin{array}{l} 1.\ \sim\sim P \\ \hline \therefore\ P \end{array}$$

4. Provide a deduction for the argument in Problem 3 using only R and **ID**.

5. Provide a deduction for the following argument, using only R and **ID**.

$$\begin{array}{l} 1.\ \ P \\ \hline \therefore\ \sim\sim P \end{array}$$

6. Suppose we use a stripped-down deductive system – call it "**SDS 1**" – containing only **ID** and the rules **R**, ~+, ~**–**, ∨**–**, ∧**–**, and **Inward DM**.[1]

Inward DM (In DM)

$$\frac{\sim(\bullet \vee \blacktriangle)}{(\sim\bullet \wedge \sim\blacktriangle)} \qquad \frac{\sim(\bullet \wedge \blacktriangle)}{(\sim\bullet \vee \sim\blacktriangle)}$$

Show that **SDS1** is deductively equivalent to the Chapter Two system, by constructing an **SDS 1** deduction for each of the following two arguments.

$$\frac{\begin{array}{l}1.\ P\\ 2.\ Q\end{array}}{\therefore\ (P \wedge Q)} \qquad \frac{1.\ P}{\therefore\ (P \vee Q)}$$

7. **SDS 2** is another stripped-down system, consisting in **ID** and the rules **R**, ~+, ~, ∨+, ∧+, and **Outward DM**.

Outward DM (Out DM)

$$\frac{(\sim\bullet \wedge \sim\blacktriangle)}{\sim(\bullet \vee \blacktriangle)} \qquad \frac{(\sim\bullet \vee \sim\blacktriangle)}{\sim(\bullet \wedge \blacktriangle)}$$

Show that **SDS2** is deductively equivalent to the Chapter Two system, by constructing an **SDS 2** deduction for each of the following two arguments.

$$\frac{\begin{array}{l}1.\ (P \vee Q)\\ 2.\ \sim P\end{array}}{\therefore\ Q} \qquad \frac{1.\ (P \wedge Q)}{\therefore\ P}$$

[1] As Problems 4 and 5 show, we could also strip out ~+ and ~**–** in this system without loss of deductive power.

8. Semantics or deduction show the following rule to be valid.[2]

Double Disjunction (DD)

$$\frac{\begin{array}{c}(\bullet \vee \blacktriangle) \\ (\bullet \vee {\sim}\blacktriangle)\end{array}}{\bullet}$$

Provide a deduction for each of the following two arguments using only ∨+ and **DD** (but without using **ID**).

$$\frac{\begin{array}{l}1.\ (P \vee Q) \\ 2.\ {\sim}P\end{array}}{\therefore Q} \qquad \frac{1.\ P}{\therefore P}$$

[2] This argument form was encountered earlier, in 2.38.1 Problem A.

8. Semantics or deduction also show the following rule to be valid.[3]

Generalized Double Disjunction (GDD)

(● ∨ ▲)
(● ∨ ♥)
~(▲ ∧ ♥)
———
●

Show that Double Disjunction is derivable through GDD, by providing a deduction for the following argument using only **~–**, **∧–**, **GDD**, and **ID**.[4]

1. $(P \lor Q)$
2. $(P \lor {\sim}Q)$

$\therefore\ P$

(*Hint: first prove "~(Q ∧ ~Q)"*)

9. Provide a deduction for the following argument, using only **R**, **∨+**, **GDD**, and **ID**.

1. P
2. Q

$\therefore\ (P \land Q)$

[3] This is a variant on the rule of **Separation of Cases** (or **Disjunctive Dilemma**), discussed in 3.14.1.
[4] This could also be shown with R instead of ~–, using the strategy from Problem 4, above.