3.6.1. Leading Principle and Validity Problem

A. Note that, whether or not an argument is valid, adding its leading principle as a (further) premise always yields a (new, bigger) argument which is valid

> (**I**) **Adding** an argument's **leading principle** as a premise always results in a (new, bigger) **valid** argument.[1]

For instance, the following familiar argument is invalid.

1. $(P \rightarrow Q)$
2. Q

$\therefore P$

But adding its leading principle "$(((P \rightarrow Q) \wedge Q) \rightarrow P)$" as a third premise yields a valid argument.

1. $(P \rightarrow Q)$
2. Q
3. $(((P \rightarrow Q) \wedge Q) \rightarrow P)$

$\therefore P$

$\therefore$	**(2)**	**(1)**		**(3)**
P	Q	$(P \rightarrow Q)$	$((P \rightarrow Q) \wedge Q)$	$\mathbf{((P \rightarrow Q) \wedge Q) \rightarrow P)}$
1	1	1	1	**1**
1	0	0	0	**1**
0	1	1	1	**0**
0	0	1	0	**1**

From the nature of a validity counterexample and a leading principle, **explain why Principle (I) is true**.

[1] As noted in (Leonard 1957: 488).

B. Recall a point noted earlier: adding a tautology as a premise to an argument never affects validity – specifically¸ if an argument is invalid, the (new, bigger) argument resulting from adding a tautology will also be invalid.[2]

> **(II) Adding a tautology as a premise does not affect validity**: if an argument is valid, then that argument with a tautology as an added premise is also valid; and if an argument is invalid, then that argument with a tautology as an added premise is also invalid.

For example, since the argument "(P → Q) . Q . ∴ P" is invalid, so is the argument that results from adding the tautology "(P → P)" as a third premise. (Valuation 3 is a validity counterexample.)

1. (P → Q)
2. Q
3. (P → P)

∴ P

∴ P	(2) Q	(1) (P → Q)	(3) ((P → P)
1	1	1	1
1	0	0	1
0	**1**	**1**	**1**
0	0	1	1

Use Principles (I) and (II) to explain why Principle (III) is true.

> **(III)** If an argument's leading principle is a tautology, then that argument is valid.

[2] Noted in 2.20.1, Problem 6.

C. In 2.42 we noted the following.

> If a **disjunction is a tautology**, then **either part** of that disjunction **follows validly from the negation of the other part**.

Now in 3.3 we recognized that the sentences “$(P \vee Q)$” and “$(\sim P \rightarrow Q)$” are logically equivalent.

P	Q	~P	**(~P → Q)**	**(P ∨ Q)**
1	1	0	**1**	**1**
1	0	0	**1**	**1**
0	1	1	**1**	**1**
0	0	1	**0**	**0**

Use this logical equivalence, along with the link between arguments and leading principles, to explain the above point about disjunction.

D. As the dual of the point in **(C)**, it was also noted in 2.42 that:

> If a **conjunction is a contradiction**, then **either part** of that conjunction **entails the negation of the other part**.

And note that in this case the **negation of** that **conjunction** will be a **tautology**.

As with **(C)**, use the link between leading principles and arguments, along with logical equivalence, to explain this point.