2.18.1. Semantic Problems: Validity

A. Translate each of the following arguments into formal language, then use **truth tables** to decide whether the argument is **valid**.

1. Letitia and Lucretia won't both enjoy the movie. Letitia won't enjoy the movie. ∴ Lucretia will enjoy the movie.

2. Letitia and Lucretia won't both enjoy the movie. Lucretia will enjoy the movie. ∴ Letitia won't enjoy the movie.

3. Kitty won't leave for Reno unless Dr. Slim does. ∴ Dr. Slim won't leave for Reno unless Kitty does.

4. We won't have both a tax cut and increased spending on Logic research. We won't have increased spending on logic research unless we have a tax cut too. ∴ We won't have increased spending on Logic research, but we will have a tax cut.

5. Neko won't eat fish unless Suki does. Suki won't eat fish unless Neko does. Either Neko won't eat fish, or Suki won't. ∴ Neither Neko nor Suki will eat fish.

6. Trixie won't win the hand unless she does so without playing an ace. ∴ Unless she plays an ace, Trixie won't win the hand.

7. Either Barbie will celebrate Rex's birthday, or she'll celebrate it without giving him new a new bowling shirt. Unless she gives him a new bowling shirt, Barbie won't celebrate Rex's birthday. ∴ Barbie will celebrate Rex's birthday and give him a new bowling shirt.

8. Either mahjongg and surfing are both legal or they're both illegal. Mahjongg isn't illegal. ∴ Mahjongg is legal and surfing is too.

9. Either Jake and Jezebel will both rejoin the cult or neither of them will. ∴ Jake won't rejoin the cult unless Jezebel does.

10. Dick won't have a Campari shooter unless Dora does. Dora won't have a Campari shooter unless Dick does. ∴ Both Dick and Dora will have a Campari shooter.

11. We're having truffles or grog, but not both. Either we aren't having truffles or we're having neither truffles nor grog. ∴ We're having grog without having truffles.

12. Either Barbie and Rex will both weekend in Palm Springs or neither of them will. Either Barbie or Rex will fail to weekend in Palm Springs. ∴ Either Barbie will weekend in Palm Spring or Rex will, but not both.

(*Hint: for* 13 *through* 15, *see the remarks on relative clauses and negations in* 2.10 §3.)

13. Jack isn't a bird that can fly. ∴ Either Jack isn't a bird, or Jack can't fly.

14. Jack isn't a bird that can fly. ∴ Jack isn't a bird unless he's one that can't fly.

15. Unless Dr. Slim is a physician, he's not a physician who performs surgery. Dr. Slim's not a physician. ∴ Dr. Slim doesn't perform surgery.[1]

16. Unless Dr. Slim is a physician who performs surgery, he's not a physician. Dr. Slim performs surgery. ∴ Dr. Slim is a physician.

17. Unless it's sunny, Barbie took her umbrella. She went out without taking her umbrella. ∴ It's sunny.

18. Jack will complain unless we're not having tuna for dinner. Neko will complain unless we are having tuna for dinner. ∴ Either Jack will complain or Neko will.

19. Unless both Neko and Jack are going to Logicpalooza, Rex isn't going. Neko isn't going to Logicpalooza, but either Jack or Rex is. ∴ Jack is going to Logicpalooza.

[1] See 2.20.1 Problem 6 for further discussion.

B. Truth and Validity Puzzle. Suki, Neko, and Jack are on trial, and testify as follows.

> **Suki**: Jack isn’t guilty, but Neko is.
> **Neko**: Suki isn’t guilty unless Jack is.
> **Jack**: I’m not guilty, but either Suki or Neko is.

Use truth tables to answer the following questions.

1. If everyone told the truth, who’s guilty and who’s not?
2. If no one is guilty, who gave false testimony?
3. If everyone is guilty, who gave false testimony?
4. One person’s testimony follows validly from someone else’s – whose?

(*Adapted from* Kleene 1967: 67, Problem 14.2*; attributed to Jerome Keisler*)

C. Truth and Validity Puzzle Revisited. Suppose Suki, Neko, and Jack instead testify as follows.

> **Suki**: Jack isn’t guilty unless Neko is.
> **Neko**: Suki isn’t guilty, but Jack is.
> **Jack**: I’m not guilty unless both Suki and Neko are.

Use truth tables to answer the following questions.

1. If everyone gave false testimony, who’s guilty and who’s not?
2. If no one is guilty, who gave false testimony?
3. If everyone is guilty, who gave false testimony?
4. One person’s testimony follows validly from someone else’s – whose?

D. Compare the answers to **Problem B** with the answers to **Problem C**. Can we **explain their similarities and differences**?

(*See* 2.33 *and* 2.34 *for further discussion.*)

E. Validity and Contradictory Conclusions. Call two sentences "**contradictory**" if one is the negation of the other – for example, "P" and "~P".

Explain why **a consistent set of premises can't entail contradictory conclusions**. (In other words: explain why, **if both the following arguments are valid, Set of Sentences S must be inconsistent**.)

Set of Sentences S	Set of Sentences S
∴ P	∴ ~P

(*Hint: if Set of Sentences S is consistent, why is a validity counterexample for one of the arguments guaranteed?*)

F. **Validity and Tautologies**. Explain why a tautology only entails tautologies (that is: why the only kind of sentence that follows validly from a tautology is a tautology).

G. **Validity and Contradictory Conclusions, Revisited**. Explain why entailing two contradictory conclusions is the hallmark of an inconsistent set of sentences – something **all and only** the inconsistent sets do.

Exercise E makes half of this case already, showing that entailing contradictory conclusions is something **only** an inconsistent set of sentences can do.

> If a contradictory pair of sentences follows validly from Set of Sentences S, then S is inconsistent.

Now strengthen that claim by explaining why **every** inconsistent set of premises entails contradictory conclusions.

> If Set of Sentences S is inconsistent, then two contradictory sentences are bound to both follow validly from S.

H. **Self-Negations**. Explain why each of the following sentences are true.

1. Only a tautology follows validly from its own negation.

2. Only a contradiction entails its own negation.

(*See* 2.34.1 Problem A *for further discussion.*)