# 3.7. Conditional Semantics: Puzzles and Peculiarities

> "It is told, for instance, of Geradas, a very ancient Spartan, that, being asked by a stranger what punishment their law had appointed for adulterers, he answered, 'There are no adulterers in our country.' 'But,' replied the stranger, 'suppose there were?' 'Then,' answered he, 'the offender would have to give the plaintiff a bull with a neck so long as that he might drink from the top of Taygetus of the Eurotas river below it.' The man, surprised at this, said, 'Why, 'tis impossible to find such a bull.' Geradas smilingly replied, ''Tis as possible as to find an adulterer in Sparta.'"
>
> – Plutarch on Lycurgus, in **The Lives of the Noble Grecians and Romans**, Vol. I pp. 66-67

So far our conditional semantics has triumphed over all alternatives, in terms of getting right the facts about entailment and meaning. But here we pause to note some puzzles and potential limitations of our semantic rule for conditionals.

**1. Four Unintuitive Arguments Revisited**. Recognizing that every argument has a **leading principle**, we revisit four types of valid arguments surveyed earlier.[1]



***First***, any sentence follows validly from itself. So the following argument is valid.

1. P

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∴ P

That means the leading principle of this argument must be a tautology. However, when we recognize that the leading principle is "(P → P)," that information poses something less than a surprise.

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[1] In 3.19.

***Second*, adding premises to a valid argument always yields a valid argument**. So, the following familiar argument is valid.

$$
\begin{array}{l}
1.\ (P \vee Q) \\
2.\ \sim P \\
\hline
\therefore\ Q
\end{array}
$$

And that means this next argument is valid as well.

$$
\begin{array}{l}
1.\ (P \vee Q) \\
2.\ \sim P \\
3.\ X \\
\hline
\therefore\ Q
\end{array}
$$

In terms of conditionals: since "$(((P \vee Q) \wedge \sim P) \rightarrow Q)$" is a tautology, the conditional "$((((P \vee Q) \wedge \sim P) \wedge X) \rightarrow Q)$" is also a tautology.

More generally: **if a conditional is true in a given valuation, conjoining new sentences to the antecedent yields a bigger conditional also true in that valuation**. For example: any valuation making "$(P \rightarrow Q)$" true also makes "$((P \wedge X) \rightarrow Q)$" true.

| **P** | **Q** | **X** | **$(P \rightarrow Q)$** | **$(P \wedge X)$** | **$((P \wedge X) \rightarrow Q)$**[2] |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 | 1 |

[2] Careful reading that last truth table: "$((P \wedge X) \rightarrow Q)$" is only false in the third valuation, where the antecedent "$(P \wedge X)$" is true but the consequent "Q" is false.

Conjoining more sentences to the antecedent can only change the conditional by making it true in more valuations – in the limit case, where the consequent is added, making the sentence a tautology. For instance, the conditional "$(((P \wedge X) \wedge Q) \rightarrow Q)$" is a tautology.

***Third***: **a tautology follows validly from any premises**. The tautology "$(\sim P \vee P)$" follows validly from "X".

1. X

∴ $(P \vee \sim P)$

| | **(1)** | | **∴** |
|---|---|---|---|
| P | **X** | ~P | **(P ∨ ~P)** |
| 1 | **1** | 0 | **1** |
| 1 | 0 | 0 | 1 |
| 0 | **1** | 1 | **1** |
| 0 | 0 | 1 | 1 |

The leading principle for this argument is therefore a tautology.

| P | X | ~P | (P ∨ ~P) | **(X → (P ∨ ~P))** |
|---|---|---|---|---|
| 1 | 1 | 0 | 1 | **1** |
| 1 | 0 | 0 | 1 | **1** |
| 0 | 1 | 1 | 1 | **1** |
| 0 | 0 | 1 | 1 | **1** |

In general: **whenever the consequent is a tautology, the whole conditional is as well**. If it's true that "Rex will either pass Chemistry or he won't," it's also true that "If your house is on fire, then Rex will either pass Chemistry or he won't".

***Fourth***, **any conclusion follows validly from inconsistent premises**. For instance, the argument "$(P \wedge \sim P)$ ∴ X" is valid.

1. $(P \wedge \sim P)$

∴ X

| | | | **(1)** | **∴** |
|---|---|---|---|---|
| P | X | ~P | **(P ∧ ~P)** | **X** |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 |

So likewise **a conditional whose antecedent is a contradiction is a tautology**: since there are no valuations making the antecedent true, there are no valuations making the antecedent true and consequent false.

***

We noted earlier that an argument with inconsistent premises was bound to be valid. For to say that the premises are inconsistent means: there is no valuation making all those premises true. But in that case there is, all the moreso, no valuation making all the premises true and the conclusion false, hence no validity counterexample.

So, for instance, the following argument is valid.

1. P

2. ~P

---

∴ Q

| **(1)** | | **(2)** | ∴ |
|---|---|---|---|
| P | Q | ~P | Q |
| 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 |

But the leading principle of this valid argument illustrates that **a conditional with a contradictory antecedent is bound to be true**.

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**1. Conditional Entailments**. We noted earlier that the conditional "(P → Q)" is logically equivalent to "(~P ∨ Q)" – and that English examples suggest this is, intuitively, the right result.

**P**: The picnic will be cancelled
**Q**: It rains

"The picnic won't be cancelled unless it rains": **(~P ∨ Q)**

“The picnic will be cancelled only if it rains”: **$(P \rightarrow Q)$**

| P | Q | ~P | **$(\sim P \vee Q)$** | **$(P \rightarrow Q)$** |
|---|---|---|---|---|
| 1 | 1 | 0 | **1** | **1** |
| 1 | 0 | 0 | **0** | **0** |
| 0 | 1 | 1 | **1** | **1** |
| 0 | 0 | 1 | **1** | **1** |

But note that – since a disjunction is true as long as at least one of its two parts is true – “(~P ∨ Q)” follows validly from “~P,” and likewise from “Q”.

| P | Q | **~P** | **(~P ∨ Q)** |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | **1** | **1** |
| 0 | 0 | **1** | **1** |

| P | **Q** | **(~P ∨ Q)** |
|---|---|---|
| 1 | **1** | **1** |
| 1 | 0 | 0 |
| 0 | **1** | **1** |
| 0 | 0 | 1 |

And – being equivalent to “(~P ∨ Q)” – “(P → Q)” likewise follows validly from “~P” and from “Q”.

| P | Q | **~P** | **(P → Q)** |
|---|---|---|---|
| 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | **1** | **1** |
| 0 | 0 | **1** | **1** |

| P | **Q** | **(P → Q)** |
|---|---|---|
| 1 | **1** | **1** |
| 1 | 0 | 0 |
| 0 | **1** | **1** |
| 0 | 0 | 1 |

Yet those results are less than intuitive. For they count each of the following arguments as valid.

**Argument (1)**

1. It’s not Sunday.

∴ If it’s Sunday, then your house is on fire.

**Argument (2)**

1. Socrates is Greek.

∴ If cats are invisible, then Socrates is Greek.

[Medieval versions of these arguments?]