5.8.1. Quantifier Semantics Problems: Instances

A. For each of the numbered sentences below, state which are instances of the following universal sentence.

$$\mathbf{\forall x}\ ((Gx \land Hx) \rightarrow \exists x \sim Gx)$$

1. $((GA \land HA) \rightarrow \exists x \sim Gx)$
2. $((GA \land HA) \rightarrow \exists x \sim GA)$
3. $\forall x\, ((Gx \land Hx) \rightarrow \sim GA)$
4. $((GB \land HB) \rightarrow \exists x \sim Gx)$
5. $((GA \land HB) \rightarrow \exists x \sim Gx)$
6. $((GA \land Hx) \rightarrow \exists x \sim Gx)$

B. For each of the numbered sentences below, state which have "**$(GA \land HB)$**" as an instance.

1. $\exists x\, (GA \land Hx)$
2. $\exists x\, (Gx \land HB)$
3. $\exists x\, (Gx \land Hx)$
4. $(\exists x\, GA \land HB)$
5. $\exists x\, (GA \land HB)$

C. Based on your answer to **(A)**, state whether the universal sentence "$\mathbf{\forall x}\ ((Gx \land Hx) \rightarrow \exists x \sim Gx)$"is true or false in the following model.

A: Neko **G**__: is a cat
B: Rex **H**__: is fat

$\mathbb{D}$: {**Neko**, **Rex**}

A: **Neko** **G**: {**Neko**}
B: **Rex** **H:** {**Neko, Rex**}

D. For each of the existential sentences picked in your answer to **(B)**, state whether that sentence is true or false in the above model.

E. According to our definition of "instance," is "GA" an instance of "$\exists x\, GA$"?

F. We noted that the sentence "**∃x (Gx ∧ ∃x ~Gx)**" would, intuitively, be true wherever there were at least two objects, one G and one non-G. And in fact we can establish semantically that this sentence is logically equivalent to "(**∃x Gx ∧ ∃x ~Gx)**" by showing that each sentence entails the other.

Consider what a validity counterexample for each argument would look like.

1	(∃x Gx ∧ ∃x ~Gx)	**1**	∃x (Gx ∧ ∃x ~Gx)
0	∴ ∃x (Gx ∧ ∃x ~Gx)	**0**	∴ (∃x Gx ∧ ∃x ~Gx)

For the argument on the left it's especially easy to see the problem: to make the premise "(∃x Gx ∧ ∃x ~Gx)" true a model will need one object in the extension of "G" (to make "∃x Gx" true) and a second object not in the extension of "G" (to make "∃x ~Gx" true).

𝔻: {2, 3}

A: 2 **G**: {2}
B: 3

But the conclusion "∃x (Gx ∧ ∃x ~Gx)" has two instances in such a model.

1 (GA ∧ ∃x ~Gx)
0 (GB ∧ ∃x ~Gx)

Since "(GA ∧ ∃x ~Gx)" is true here, the model makes the conclusion "∃x (Gx ∧ ∃x ~Gx)" true, and so isn't a validity counterexample. And no modification of the model will change this: **(i)** simple replacing A with B in the extension of "G" will make the second instance "(GB ∧ ∃x ~Gx)" true, leaving the conclusion with a true instance. Putting both objects either **(ii)** in the extension of "G" or **(iii)** outside the extension of "G" will make either "∃x ~Gx" or "∃x Gx" false, and so make the premise false. And **(iv)** leaving the objects as they are but adding more objects leaves the premise and conclusion true.

Provide a similar **semantic explanation for why the right argument** must likewise be **valid**.

G. Return once more to the model where Neko is a cat and Rex isn't one.

G__: is a cat

$\mathbb{D}$: {**Neko**, **Rex**}

A: **Neko** **G**: {**Neko**}
B: **Rex**

The discussion of instances noted that if our account of "instance" involved replacing **every** occurrence of "x" in the scope formula (whether free or not), then we wrongly count the consistent existential sentence "**∃x (Gx ∧ ∃x ~Gx)**" a contradiction.

But suppose a critic replies that we should instead count as instances **both** the sentences following the 'only free variables' condition **and** those ignoring that condition. By that more relaxed standard "**∃x (Gx ∧ ∃x ~Gx)**" would have four instances in this model.

(i) **(GA ∧ ∃x ~GA)** (iii) **(GB ∧ ∃x ~GA)**
(ii) **(GA ∧ ∃x ~GB)** (iv) **(GB ∧ ∃x ~GB)**

Since "**∃x (Gx ∧ ~Gx)**" still has at least one true instance in this model – Sentences (ii) and (iii) – it is rightly not counted as a contradiction on this account of instances.

Show that this more lax standard for being an instance leads to incorrect results, using as test case the following universal sentence.

∀x (Gx → ∃x ~Gx)

G__: __is a cat

For every object: if that object is a cat, then there's some object which isn't a cat.

On our account of instances, the scope formula "(Gx → ∃y ~Gy)" has two instances in our model (repeated here).

G__: is a cat

𝔻: {**Neko**, **Rex**}

A: **Neko** **G**: {**Neko**}
B: **Rex**

Instances of "∀x (Gx → ∃x ~Gx)" in this model:

(1) (GA → ∃x ~Gx)
(2) (GB → ∃x ~Gx)

To see that both of these conditionals are true in this model, it suffices to note that the consequent "∃x ~Gx" is true in this model – for there is indeed an object (Rex) which isn't a cat. But conditional semantics dictates that the whole **conditional is true whenever its consequent is true**.

●	▲	(● → ▲)
1	**1**	**1**
1	0	0
0	**1**	**1**
0	0	1

That makes sense intuitively: in a situation where at least one object is non-G, it will be true of any object we pick that if it's G, something *isn't G*.

But according to the more lax alternative account of instances, the sentence "∀x (Gx → ∃x ~Gx)" will have four instances in this model.

☠ **Instances of "∀x (Gx → ∃x ~Gx)" in this model?** ☠

(1) (GA → ∃x ~Gx) (3) (GA → ∃x ~GA)
(2) (GB → ∃x ~Gx) (4) (GB → ∃x ~GB)

Will all these be true in this model?