

4.8. Syllogistic Deduction Extended

1. Tautology and Contradiction. Previously we extended categorical sentences beyond the traditional categorical form in allowing the same term to appear in both the subject and predicate positions. For an existence claim – e.g., “G’s exist” or “There are G’s” – was stated as “Some G are G”.

Likewise, indirect proofs used an ‘absurd’ sentence serving as the categorical counterpart to a contradiction in sentence logic: a sentence of the form “Some G are non-G”.

This ‘extended categorical form’ also allows tautologies (logical truths) – sentences is guaranteed to be true, regardless of which term appears in it. A tautology in categorical form will be a sentence of the form “All G are G”.

And just as in sentence logic, tautologies will be theorems – sentences **provable** in the deductive system, without appeal to any premises.

- | | | |
|----|---|-----------------------|
| | | Get: All G are G (ID) |
| 1. | <div style="border: 1px solid black; padding: 2px; display: inline-block;">Some G are non-G</div> | AID |
| 2. | All G are G | 2, ID |

2. One-Predicate Existentials and Universals. Consider: every object in the universe is either G or non-G. And that holds regardless of what predicate G translates: every object is either a cat or a non-cat, but likewise every object is either a unicorn or a non-unicorn. So between the Gs and the non-Gs, we include every object in the universe.

That means that if both of the following claims are true, then **everything is H**.

- (1) All G are H
- (2) All non-G are H.

Of course that holds for any predicate, not just for H. So if both the following claims are true, then **everything is J**.

- (3) All G are J
- (4) All non-G are J.

But importantly, this holds likewise for predicate G itself: if all G are G, and all non-G are G, then **everything is G**.

- (5) All G are G
- (6) All non-G are G

And in this case we can simplify our phrasing. For as already noted, the first claim, “All G are G,” is a tautology built into the logical system. The only claim of substance here is thus the second: that “All non-G are G”. So here is our way of making a one-predicate universe claim in (extended) categorical form.

“Everything is G”: All non-G are G

If we wish to make a one-predicate universal claim about a negative predicate, we can begin with the same two sentences (or sentence-forms).

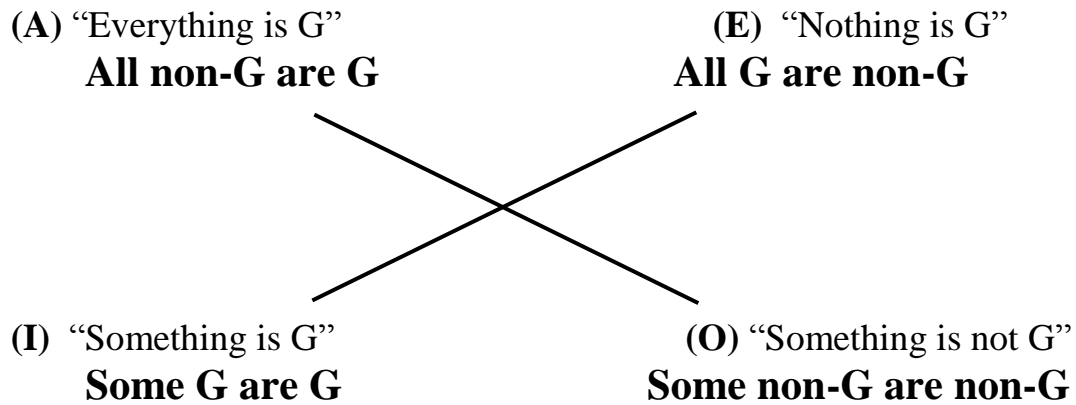
- (7) All G are non-G
- (8) All non-G are non-G

In this case the second sentence is the tautology, meaning all the content is carried by the first. So to say “Everything is non-G” in categorical form, we write “All G are non-G”.

“Everything is non-G” (equivalently: “Nothing is G”):
All G are non-G

That allows us to make a one-predicate counterpart to the tradition square of opposition.

One-Predicate Square of Opposition



As an application of this one-predicate sentence format, we can show that if all Gs are H, and all Gs are also non-H, then **there are no Gs** (that Gs don't exist).

- | | | |
|----|------------------|---------------------------|
| 1. | All G are H | |
| 2. | All G are non-H | |
| 3. | Some G are G | Get: All G are non-G (ID) |
| 4. | Some G are H | AID |
| 5. | Some H are G | 1, 3, L |
| 6. | Some H are non-H | 4, S |
| 7. | All G are non-G | 2, 5, Lim L |
| | | 3, 6, ID |

We add the following intuitively valid argument form as a further rule of the deductive system.

Existence (E)

1. Some men are mammals.

∴ Men exist.

1. Some G are H

∴ Some G are G

1. Some men are non-unicorns.

∴ Men exist.

1. Some G are non-H

∴ Some G are G

While this rule allows us to deduce an existence claim for the subject of an I or O sentence, it's trivial to deduce an existence claim for the predicate as well. (In fact, no Indirect Deduction is needed for this.)

1. Some G are H

2. Some H are G	1, Sw
3. Some H are H	2, E

Get: Some H are H

And (E) yields a deduction for the following argument.

1. Unicorns don't exist.

∴ No horses are unicorns.

All G are non-G.

∴ **All H are non-G.**

1. All G are non-G

2. Some H are G	Get: All H are non-G (ID)
3. Some G are H	AID
4. Some G are G	2, Sw
5. Some G are non-G	3, E
	1, 4, L

6. All H are non-G 2, 5, ID

[A further derived rule:

Repetition (for universal and existential sentences)

- | | | |
|-------|---------------------|------------------|
| 1. | All G are H | |
| <hr/> | | |
| | | Get: All G are H |
| 2. | All non-H are non-G | 1, S |
| 3. | All G are H | 2, S |

- | | | |
|-------|--------------|-------------------|
| 1. | Some G are H | |
| <hr/> | | |
| | | Get: Some G are H |
| 2. | Some H are G | 1, S |
| 3. | Some G are H | 2, S |