5.13.1. Quantifier Deduction Problems

A. **Translate** each of the following English arguments into the formal language of Chapter Four, then show that the argument is valid by constructing a **deduction** of it.

1a. If anything is material then everything is. $\therefore$ It's not the case that only some things are material. (*Can be deduced without ∀– or ∃–.*)

1b. It's not the case that only some things are material. $\therefore$ If anything is material then everything is. (*Can be deduced without ∀– or ∃–.*)

[1a. $\sim(\exists x\ Gx \land \exists x \sim Gx)\ \therefore\ (\exists x\ Gx \rightarrow \forall x\ Gx)$
1b. $(\exists x\ Gx \rightarrow \forall x\ Gx)\ \therefore\ \sim(\exists x\ Gx \land \exists x \sim Gx)$]

2. Only millionaires are club members. No philosophers are millionaires. Rex is a philosopher. $\therefore$ Rex is not a club member. (*Can be deduced without ID.*)

3. All monoids are semigroups, but not all semigroups are monoids. $\therefore$ It's not the case that: something is a monoid if and only if it's a semigroup.

B. Derived Rule Problems

1. The rule of Quantifier Negation comes in Inward and Outward forms.

Inward Quantifier Negation (In-QN)

$$\frac{\sim\forall x\ \bullet}{\exists x\ \sim\bullet} \qquad \frac{\sim\exists x\ \bullet}{\forall x\ \sim\bullet}$$

Outward Quantifier Negation (Out-QN)

$$\frac{\exists x\ \sim\bullet}{\sim\forall x\ \bullet} \qquad \frac{\forall x\ \sim\bullet}{\sim\exists x\ \bullet}$$

Show that we can treat **Outward QN** as a **derived rule** in the Chapter Five deductive system, by building **deductions for arguments (1a) and (1b) using only** $\forall$**–** , $\exists$**–**, ~**–**, **and Indirect Deduction**.

(1a)	**(1b)**
1. $\exists x \sim Gx$	1. $\forall x \sim Gx$
$\therefore \sim\forall x\ Gx$	$\therefore \sim\exists x\ Gx$

2a. Suppose we call the following rule **Existential Introduction**.

Existential Introduction ("E-Intro") (∃+)

$$\frac{\bullet_I}{\therefore\ \exists x \bullet}$$

where $\bullet_I$ is an **instance** of the scope formula $\bullet$

∃+ is a derived rule of the Chapter Five deductive system, since any argument fitting this inference pattern can be deduced using only the existing deductive rules of Chapter Five. Show that all of the following arguments are deducible in the Chapter Five system.

$$\frac{1.\ (GA \land HA)}{\therefore\ \exists x\,(Gx \land Hx)} \qquad \frac{1.\ (GA \land HA)}{\therefore\ \exists x\,(Gx \land HA)} \qquad \frac{1.\ (GA \land HA)}{\therefore\ \exists x\,(GA \land HA)}$$

2b. Explain the **mistake** in the following deduction.

☠ A Bad Deduction ☠

1.	$(GA \land \sim GB)$	
		Get: $\exists x\ (Gx \land \sim Gx)$ (ID)
2.	$\sim\exists x\ (Gx \land \sim Gx)$	AID
3.	$\forall x \sim(Gx \land \sim Gx)$	2, QN
4.	$\sim(GA \land \sim GB)$	3, ∀−
5.	$(GA \land \sim GB)$	1, R
6.	$\exists x\ (Gx \land \sim Gx)$	2, 4, 5, ID

2c. Suppose we remove the rule ∀**–** from the deductive system and replace it with the rule **∃+** (from 2a, above). In this new deductive system, **∀– can be treated as a derived rule**. Show, for instance, that the following argument is deducible in this new system.

1. $\forall x\ Gx$

$\therefore$ GA

2d. The following argument is clearly **invalid**. (A situation with only human Rex and cat Neko serves as a counterexample).

☠ An Invalid Argument ☠

1. Everything is either a cat or a human.

$\therefore$ Either Rex is a cat or Neko is a human.

Explain the **mistake** in the following deduction for that argument.

☠ A Bad Deduction ☠

1.	$\forall x\,(Gx \vee Hx)$	
		Get: $(GA \vee HB)$ (ID)
2.	$\sim(GA \vee HB)$	AID
3.	$\exists x \sim(Gx \vee Hx)$	2, **∃+**
4.	$\sim\forall x\,(Gx \vee Hx)$	3, QN
5.	$\forall x\,(Gx \vee Hx)$	1, R
6.	$(GA \vee HB)$	2, 4, 5, ID