

❖ *Quantifiers and Variables: Translation and Semantics* ❖

5.4. Quantifiers and Variables

Quantifiers such as “all” and “some” are the last new bit of logical form. But as we’ll see, quantified sentences turn out to be the most complex examples of logical form so far. So we begin by sharpening our understanding of them through simple English examples of what quantifiers are – and what they’re not.

1. What Quantifiers Are Not: Names Revisited. Having mastered translation of proper names into the formal language, it’s tempting to sweep other phrases under the umbrella of proper names by translating them the same way. The following two sentences, for instance, seem nicely parallel.¹

- (1) **Methods of Logic** is an interesting book.
- (2) **Something** is an interesting book.

With “something” filling the grammatical subject position of (2), just as the proper name “Methods of Logic” does in (1), it could seem sensible to translate “something” by a name letter.

But the parallel breaks down on further analysis.

- (1) Methods of Logic is an interesting book.
- (3) Methods of Logic is interesting, and Methods of Logic is a book.
- (2) Something is an interesting book.
- (4) Something is interesting, and something is a book.

With proper names we made a practice of translating ‘stacked up’ predicates – like “is an interesting book” – as a conjunction in disguise. That treats (1) and (3) as equivalent in logical meaning. And that looks like the right result: (1) and (3) mean the same thing; and whenever one of these sentences is true, the other is as well.

¹ Adapting an example from (Quine 1959: 83-84).

But (2) and (4) aren't true in exactly the same situations.

(2) Something is an interesting book.

(4) Something is interesting, and something is a book.

In a situation containing just two objects – an interesting speaker and a boring book – (4) would be true, but (2) would be false.

If that seems too subtle, the next two sentences draw the same distinction more boldly.

(5) Something is a non-feline feline.

(6) Something is non-feline, and something is feline.

What (6) reports certainly seems possible – indeed, the actual world is a situation where something is non-feline (say, the book *Methods of Logic*) and something is feline (say, Neko). But what Sentence (5) reports seems impossible. (5) looks like a flat-out contradiction, false in every possible situation (including the actual one). Hence the actual world illustrates that (5) and (6) **aren't** true in all the same situations; and predictably, the two sentences don't mean the same thing.

By comparison: Sentences (7) and (8), using the proper name “Rex,” do seem logically equivalent. (They both look like contradictions.)

(7) Rex is a non-feline feline.

(8) Rex is non-feline, and Rex is feline.

This illustrates that a quantifier such as “something,” even when appearing in the subject position of an English sentence, doesn't behave logically like a proper name. That's why we resist translating English quantifiers as name letters.

(We could make the same case in terms of truth and validity: given the obvious importance of truth and falsehood to validity, and the clear differences in truth and falsehood between Sentences (5) and (6), our formal language had better have a way of showing the difference between such sentences – when a sentence is talking about the same object(s) throughout, like (5), and when not, like (6). But as long as we translate quantifiers as

names, that difference will be papered over – as examples such as (7) and (8) make clear. So again: we resist translating quantifiers as names.)

2. Quantifiers and Variables. Further examples steer us toward a proper translation of sentences with quantifiers. Suppose, first, that a materialist philosopher of mind makes the following **universal** claim.

(9) **Everything in the universe is a physical object.**

The predicate “is a physical object” is translated by a predicate letter.

G: ____ is a physical object

We still lack a way of talking formally about **every** object in the universe. But a somewhat wordy rephrasing of Sentence (9) offers a clue.

(10) **For every object in the universe, the following holds true of it:**
it is a physical object.

Put this way, quantified sentence (9) has two components: (i) the mini-sentence “it is a physical object” on the right, and (ii) a quantifier phrase applying that mini-sentence to every object in the universe.

Now the mini-sentence “it is a physical object” resembles familiar English sentences such as “Rex is a physical object” which pose no obstacle to translation. And the word “it” does act in many respects like the proper name “Rex”: both words can fill the blank in a predicate, yielding a sentence of English; and both serve to refer to some object. So here again we may be tempted to translate the term like a proper name.

But there’s an important difference here as well. The name “Rex” is a **proper** name precisely because it always points to the same object. By contrast, the word “it” refers to different things from one sentence to the next. I can point to the Lincoln Log Cabin and say “It is made of wood,” then point to the Cathedral of Learning and say “It is not made of wood”. In so doing I have uttered no contradiction – since “it” in the two sentences pointed at different things. By contrast, if I say “Suki is from Pennsylvania”

and “Suki is not from Pennsylvania” I have contradicted myself – since a proper name like “Suki” refers consistently to the same object.

Because “it” can vary in what it’s pointing to, the word only refers successfully with outside help – a pointing finger, or a context where one especially point-worthy object stands out. By varying the context or target of the finger, we vary the reference of “it”. By contrast, the proper name “Suki” refers to Suki even when she’s well out of eyeshot.

For that reason an “it” sentence of English without such outside help **fails to make a complete claim**. If you find a scrap of paper in a field with the sentence “It is from Pennsylvania” written on it, you don’t know what claim is being made – not even if you know the geographical origin of every object in the world. But a scrap of paper reading “Suki is from Pennsylvania” on its own expresses a complete sentence, capable of truth or falsehood. (In fact it’s false.) An “it” sentence isn’t the sort of complete-claim-maker we find in a sentence using a proper name.

To highlight these differences we resist translating “it” (and related terms like “this” and “that”) as name letters. Instead we add new pointing symbols to the formal language: lower-case letters “p” through “z”. These are the **variables**.

Variables: lower-case letters “p” through “z”

In terms of sentence construction, variables show up in the same locations as proper names: after a predicate letter. So using the previous translation key we translate the ambiguous English mini-sentence “it is a material object” as follows.

G: ____ is a physical object

It is a physical object.

Gx

But because both name letters and variables occupy the same grammatical role – filling in the blank in a predicate letter – and both serve to refer to objects, we view them as different species within a larger family – what we’ll call the “terms”.

Term: any name letter or variable

Variables and predicate letters take our universal sentence part-way into formal translation.

For every object in the universe, the following holds true of it:
it is a physical object

For every object, x, the following holds of x: Gx

We then need formal symbols for universal terms like “every” and “all”. We will express universal terms formally by the **universal symbol** “ \forall ”. (When reading this symbol aloud, it’s pronounced “universal”.)

Combining a universal symbol with a variable yields a **universal quantifier**. For instance, “ $\forall x$ ” means “for every object, x, the following holds of x”.

Now we can translate the entire English sentence.

G: ____ is a physical object.

(10) Everything in the universe is a physical object.

(For every object, x, the following holds of x: x is a physical object)

(11) $\forall x Gx$

Next, suppose a dualist philosopher of mind disputes this materialist claim, holding instead that while some things in the universe are physical objects (tables and chairs), others (particularly minds or souls) are not.

**(12) Some things in the universe are physical objects,
 but some things in the universe are not physical objects.**

That dualist claim is a conjunction. The left half is

(12a) Some things in the universe are physical objects

while the right half is

(12b) Some things in the universe are not physical objects.

Following a tradition reaching back to Aristotle, we interpret “**some**” to mean: **at least one**.²

Rephrasing each of these sentences in the same wordy fashion as before, the left sentence reads as follows.

(12a) Some things in the universe are physical objects

For some (at least one) object, x , the following holds of x :
 x is a physical object

Employing the same translation key, the translation begins like this.

G: ____ is a physical object.

For some (at least one) object, x , the following holds of x : Gx

As a formal means of expressing “some,” we introduce the **existential symbol** “ \exists ” (pronounced “existential”). Like the universal, the existential symbol combines with a variable to make an **existential quantifier**.

We then translate the left sentence in the dualist conjunction like so.

(12a) Some things in the universe are physical objects

(13a) $\exists x Gx$

² We make this assumption for a variety of (related) reasons involving, among others, simplicity and duality. How to address the cases where this assumption differs from conversational English is addressed in our later discussion of pragmatics, in 7.x.

And the dualist's claim in its entirety translates as follows.³

- (12) Some things in the universe are physical objects,
but some things in the universe are not physical objects.

$$(13) (\exists x Gx \wedge \exists x \sim Gx)$$

Finally, suppose an idealist philosopher makes the following claim.

- (14) **Nothing in the universe is a physical object.**

While we could introduce a third quantifier symbol to express “nothing,” it turns out we can express this in our formal language using symbols already on hand.

Intuitively, the sentence “Nothing in the universe is a physical object” means the same as “Not (even) one thing in the universe is a physical object”. So a “nothing” sentence can be read as the **denial of an existential sentence**.

G: ____ is a physical object.

Nothing in the universe is a physical object.

$$(15) \sim \exists x Gx$$

Interestingly, this isn't the only avenue available to us to express “nothing” in the (expanded) formal language. In fact we'll later find different, but semantically equivalent, ways of expressing all of the quantified sentences covered so far.

³ This sentence illustrates again the difference stressed earlier: that the object(s) said to be physical and those claimed to be non-physical are not here said to be the **same** object(s). One might suppose different variables are called for – say, “x” and “y” – to express that difference. But as we'll see later, in 5.6, different variables aren't in fact needed here to express that different objects are intended.

Summary: Quantifiers and Variables

- **Variables:** lower case letters **p through z** (with or without numerical subscripts). Variables are the formal counterpart to short-term, context-sensitive pointers of English such as “it”. The English sentence “**It is physical**” translates as “**Gx**” (where “G__” translates “__ is physical”)
- **Universal quantifier:** the universal symbol “**∀**” followed by a variable. The universal quantifier “**∀x**” is the formal counterpart of the English words: “For every object, the following holds of it”.

The English sentence “**Everything is physical**” – rephrased as: “For every object, the following holds of it: it is physical” – is translated into the formal language as “**∀x Gx**” (where “G__” translates “__ is physical”).

- **Existential quantifier:** the existential symbol “**∃**” followed by a variable. The existential quantifier “**∃x**” is the formal counterpart of the English words: “For some object, the following holds of it”.

The English sentence “**Something is physical**” – rephrased as: “For some object, the following holds of it: it is physical” – is translated into the formal language as “**∃x Gx**” (where “G__” translates “__ is physical”).