4.7. Indirect Deduction and Categorical Syllogism

1. Indirect Deductions. In an **indirect deduction** the conclusion is deduced from the premises not simply through a chain of valid inferences from premises to conclusion. Instead, we **assumes the opposite of the conclusion** and shows that this assumption, in combination with the premises**, leads to a logical absurdity** – a sentence which couldn't possibly be true.[1] Leading validly to such an absurdity, the assumption is then concluded not to be true.

A logically absurd sentence, incapable of being true, is the one-sentence categorical counterpart to the contradictions of sentence logic (some sentence and its negation). A categorical **contradiction** will be of the following form.[2]

Some G are non-G

If, in light of the premises, assuming the opposite of the conclusion permits us to deduce some sentence of this form, that assumption of the opposite has been shown to be false. In that case the conclusion itself must (given the premises) be true.

[1] Indirect deductions were discussed at length in *3.38. Fundamental of Indirect Deduction*. They are a deductive form of indirect arguments, first discussed in *3.21. The Semantic Test of Validity: An Indirect Approach*.
[2] As with the earlier existence assumption, our absurd sentence requires that categorical sentence form permit the same term in both subject and predicate of the sentence.

The "**opposite**" of a sentence in categorical form is its **contradictory** – the sentence diagonal from it in the Square of Opposition.

Square of Opposition

(A) "All G are H"	**(E)** "No G are H"
All G are H	**All G are non-H**
(I) "Some G are H"	**(O)** "Some G are non-H"
Some G are H	**Some G are non-H**

So if the conclusion is "All G are H," its opposite is "Some G are non-H". If the conclusion is "Some G are H," its opposite is "All G are non-H."

The following argument has already been shown valid by means of a standard deduction.

1. All G are non-H
2. All I are H

∴ All G are non-I

We now construct an indirect deduction. We first assume the **contradictory of the conclusion** – in this case, "Some G are I". As in sentence logic deductions, this assumption is called the **Assumption of the Indirect Deduction** ("**AID**" for short).

1. All G are non-H
2. All I are H

~~Get~~: All G are non-I (ID)

3. | Some G are I — AID

From here Switching and Linking lead to our absurd sentence "Some G are non-G".

1.	All G are non-H	
2.	All I are H	
		Get: All G are non-I (ID)
3.	Some G are I	AID
4.	Some G are H	2, 3, L
5.	All H are non-G	1, S
6.	Some G are non-G	4, 5, L
7.	All G are non-I	3, 6, ID

Since the AID (Line 3) has been proven to lead to an absurdity (Line 6), the AID is rejected in favor of its opposite – the desired conclusion (Line 7).

2. Indirect Deduction with Limited Rules. Since indirect deductions keep the rules of Switching and Linking, all of the previous problems can be deduced through indirect deduction. In fact, however, indirect deductions can achieve all the same results with only **half** of each rule – what we will call "**limited rules**".

Limited Switching applies **only to universal sentences** (unlike ordinary Switching, which also allowed existential sentences).

Limited Switching (LimS)

All ⬟ are ✱
―――――――
All non-✱ are non-⬟

All non-⬟ are non-✱
―――――――
All ✱ are ⬟

All ⬟ are non-✱
―――――――
All ✱ are non-⬟

All non-⬟ are ✱
―――――――
All non-✱ are ⬟

Like ordinary Linking, **Limited Linking** joins two sentences by a middle, linking premise which must be universal, and whose terms are the predicates of the other two sentences. But in Limited Linking the other premise and the conclusion must both be **existential** sentences.

Limited Linking (LimL)

	Some ● are ▲
Linking Premise ⇒	All ▲ are ✱
	Some ● are ✱

Note that ordinary deductions could not have got by with only these limited rules. For instance, Argument 1 from 2.1 Section 3 could not have been deduced using just Limited Linking.

1.	All G are H	
2.	All H are I	
		Get: All G are I (ID)
3.	Some G are non-I	AID
4.	All non-I are non-H	2, Lim Sw
5.	Some G are non-H	3, 4, Lim L
6.	All non-H are non-G	1, Lim Sw
7.	Some G are non-G	5, 6, Lim L
8.	All G are I	3, 7, ID

Likewise Argument 7 of that section could not be deduced using just Limited Switching. Here is the indirect deduction of Argument 7.

1.	All H are G	
2.	Some H are I	
		~~Get~~: Some G are I (ID)
3.	All G are non-I	AID
4.	All I are non-G	3, Lim Sw
5.	Some H are non-G	2, 4, Lim L
6.	All non-G are non-H	1, Lim Sw
7.	Some H are non-H	5, 6, Lim L
8.	All G are non-I	3, 7, ID

This demonstrates that the shift to indirect deduction adds some genuine deductive power, over the ordinary deductions without ID.

Indeed, because the deductive system, with ID added, can make all the moves the non-limited rules could, we can provide deductions of the (non-limited) remainder of each rule.

For example, Limited Switching applies only to universal sentences; and the version of Switching applying to existential sentences can be deduced through ID and limited rules.

1.	Some G are H	
		~~Get~~: Some H are G (ID)
1.	All H are non-G	AID
2.	Some G are non-G	1, 2, Lim L
3.	Some H are G	2, ID

Likewise, Limited Linking applies only to existential sentences; and we can deduce the version of Linking applying to universal sentences, using ID and limited rules.

1.	All G are H	
2.	All H are I	
		Get: All G are I (ID)
3.	Some G are non-I	AID
4.	All non-I are non-H	2, Lim Sw
5.	Some G are non-H	3, 4, Lim L
6.	All non-H are non-G	1, Lim Sw
7.	Some G are non-G	5, 6, Lim L
8.	All H are non-G	3, 7, ID

That said, we will not restrict ourselves to the limited rules in what follows. They are useful here just to illustrate that the deductive system with indirect deduction can do something which our original form of deduction cannot.

3. Contradiction: One Sentence or Two? In the indirect deductions of sentence logic, the "contradiction" which closes an ID box was a pair of sentences: some sentence and its negation. And while we don't have negations of sentence in syllogistic logic (only negations of terms), we could still think of a contradiction as a pair of sentences: some sentence, and its opposite in the Square of Opposition.

Square of Opposition

(A) "All G are H"
All G are H

(E) "No G are H"
All G are non-H

(I) "Some G are H"
Some G are H

(O) "Some G are non-H"
Some G are non-H

In that case an A and O sentence could form a two-sentence contradiction, and an I and E sentence could as well.

Two-Sentence "Contradictions":

(**A**) All men are mortal
(**O**) Some men are non-mortal

(**E**) All lizards are non-mammals
(**I**) Some lizards are mammals

So we might instead have arranged indirect deductions such that the ID box closes only when a sentence and its opposite both appear in the ID box. Would that format have yielded different results, in terms of which arguments an indirect deduction can show to be valid?

As a matter of fact, it would make no difference. For on the one hand we can show that, with a two-sentence contradiction in hand, we can deduce a one-sentence contradiction. (In fact, indirect deduction is not required to show this.)

1. All G are non-H (**E Sentence**)
2. Some G are H (**I Sentence**)

Get: Some G are non-G

3. All H are non-G (1, Lim Sw)
4. Some G are non-G (2, 3, Lim L)

1. All G are H (**A Sentence**)
2. Some G are non-H (**O Sentence**)

Get: Some G are non-G

3. All non-H are non-G (1, Lim Sw)
4. Some G are non-G (2, 3, Lim L)

And on the other hand, with a one-sentence contradiction we can deduce each half of a two-sentence contradiction. (Here ID is required.)

1.	Some G are non-G (**One-Sentence Contradiction**)	
		~~Get~~: All H are I (ID)
2.	Some H are non-I	AID
3.	Some non-G are G	1, Sw
4.	Some G are non-G	3, Sw
5.	All H are I	2, 4, ID

1.	Some G are non-G (**One-Sentence Contradiction**)	
		~~Get~~: Some H are non-I (ID)
2.	All H are I	AID
3.	Some non-G are G	1, Sw
4.	Some G are non-G	3, Sw
5.	Some H are non-I	2, 4, ID

As both of these deductions illustrate, **any sentence follows validly from a one-sentence contradiction**. For in neither deduction is the AID ever used to deduce subsequent lines. (That is: the justification of Lines 3 and 4 makes no appeal to Line 2.) Significantly: **it is not a requirement of indirect deduction that the AID play any role inside the ID box**.

With that in mind, we see that any sentence is likewise deducible from a two-sentence contradiction (this time using ID).

1.	All G are H (**A Sentence**)	
2.	Some G are non-H (**O Sentence**)	
		~~Get~~: Some I are J (ID)
3.	All I are J	AID
4.	All non-H are non-G	(1, Lim Sw)
5.	Some G are non-G	(2, 4, Lim L)
6.	Some I are J	2, 4, ID

1.	All G are non-H (**E Sentence**)	
2.	Some G are H (**I Sentence**)	
		Get: Some I are J (ID)
3.	All I are J	AID
4.	All H are non-G	(1, Lim Sw)
5.	Some G are non-G	(2, 4, Lim L)
6.	Some I are J	2, 4, ID

So it is a matter of choice whether we require an ID format using two-sentence contradictions or one-sentence contradictions. Since one-sentence contradictions are simpler (involving only one line), in what follows we retain the one-sentence contradiction format for ID.