

❖ *Proofs and Deductions* ❖

3.14. Conditional Deduction

Indirect deduction introduced us to hypothetical reasoning: deducing what *would* happen, if a certain sentence *were* accepted as true. A similar strategy underlies a method of deduction devoted solely to conditional sentences.

Here, instead of hypothetically assuming the negation of the conclusion (as in ID), we assume the *antecedent* of the conditional sentence we're interested in deducing – exploring where that antecedent would lead. If we succeed in tracing a deductive trail from the assumed antecedent to the *consequent* of the conditional, we have shown that **if the antecedent were true, the consequent would be true**. Such hypothetical reasoning thereby establishes that the conditional is true.

For obvious reasons we call this new deductive strategy **Conditional Deduction** (or “**CD**” for short).

The following is an intuitively valid argument, with conditional conclusion.

1. We're having either ice cream or cake.

(So,) If we're not having ice cream, then we're having cake.

Conditional Deduction provides a natural way of deducing this conditional conclusion from the premise.

We first restate the argument in formal language.

P: We're having ice cream

Q: We're having cake

1. $(P \vee Q)$

$\therefore (\sim P \rightarrow Q)$

The conditional deduction (CD) of the argument is set up like so.

1. $(P \vee Q)$
- Get $(\sim P \rightarrow Q)$ (CD)

Then the hypothetical reasoning begins – as with ID, marked as hypothetical by occurring within a box. First the **Assumption of the Conditional Deduction** (ACD): suppose we *did* have the antecedent.

1. $(P \vee Q)$
- Get $(\sim P \rightarrow Q)$ (CD)

 2. $\sim P$ ACD

From lines (1) and (2), “Q” follows by simple \vee –.

1. $(P \vee Q)$
- Get $(\sim P \rightarrow Q)$ (CD)

 2. $\sim P$ ACD
 3. Q 1, 2, \vee –

But having demonstrated that if “ $\sim P$ ” then “Q”, we have established the truth of the whole conditional. The hypothetical reasoning is then complete.

1. $(P \vee Q)$
- Get $(\sim P \rightarrow Q)$ (CD)

 2. $\sim P$ ACD
 3. Q 1, 2, \vee –

4. $(\sim P \rightarrow Q)$ 2, 3, CD

The conclusion of a conditional deduction is justified by citing the **assumption of the antecedent** (here, line 2), and the **consequent** deduced from that assumption (here, line 3).

Beyond their appeal to hypothetical reasoning, ID and CD are alike in other respects as well. First: as with IDs, once a CD box is closed all lines in that box become **unusable**. No rule can be applied to any line in a closed box.

And like IDs, CDs can be used **recursively**: in the midst of one CD, we can start another, hence embedding CDs within CDs. The following is a simple example.

1. We'll have either ice cream, or cake, or pie.

∴ If we don't have ice cream, then if we don't have cake we'll have pie.

1. $(P \vee (Q \vee R))$

∴ $(\sim P \rightarrow (\sim Q \rightarrow R))$

1. $((P \vee (Q \vee R))$

		Get: $(\sim P \rightarrow (\sim Q \rightarrow R))$ (CD)
2.	$\sim P$	ACD
3.	$(Q \vee R)$	1, 2, $\vee-$
		Get: $(\sim Q \rightarrow R)$ (CD)
4.	$\sim Q$	ACD
5.	R	3, 4, $\vee-$
6.	$(\sim Q \rightarrow R)$	4, 5, CD
7.	$(\sim P \rightarrow (\sim Q \rightarrow R))$	2, 6, ID

Conditional deduction is fundamentally unlike indirect deduction in one way, however: whereas ID is suitable for deducing *any* type of sentence, CD is only useful for deducing a conditional.

That point marks a change in our deductive strategy. While the advent of ID led to a default strategy of automatically reaching for ID, **for any conditional conclusion we now automatically use CD** (unless an easier way of getting that conclusion is obvious).

Summary: Conditional Deduction (CD)

- Write (**CD**) next to the “Get” line, as a reminder.
- Immediately following the “Get” line, begin a **box**, in which the Conditional Deduction occurs.
- The first line in the CD box is the **Assumption of the Conditional Deduction (ACD)**: the **antecedent** of the sentence on the “Get” line.
- Using deductive rules on all available lines (premises and ACD), deduce the **consequent** of the sentence on the “Get” line.
- Once the consequent has been deduced, **close** the CD box. (When the CD box is closed, no rules can be applied to any line in that box. These sentences become “**unusable**”.)
- Beneath the CD box write the conclusion of the argument (the sentence on the “Get” line). The justification for this conclusion cites two lines: the **ACD**, and the **consequent**. These two numbers are followed by “**CD**”.
- If the conclusion of an argument is a **conditional**, automatically use **CD** for that argument. If the conclusion of the argument is any other type of sentence (sentence letter, negation, conjunction, or disjunction), use **ID**.