Houston Chp. 1

H1.1

mass = m speed = v pressure = P

$$PV = \frac{1}{3} n N_A m \langle v^2 \rangle$$

$$P = \frac{n N_A m \langle v^2 \rangle}{3V} \quad \text{if } m_2 = \tfrac{1}{2} m, \; P_2 = \frac{P}{2}$$

$$\text{if } v_2 = 2v, \; v_2^2 = 4v^2, \; P_2 = 4P$$

overall $P_2 = \frac{4}{2} P = 2P \rightarrow \boxed{\text{(a) increase}}$

H1.8

$$F(v)\,dv = 4\pi v^2 \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) dv$$

$$\frac{dF(v)}{dv} = \underbrace{4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2}}_{\text{divide both sides by this to cancel it}} \left[\left[\exp\left(-\frac{mv^2}{2k_B T}\right)\right](2v) + (v^2)\left[\left(-\frac{mv}{k_B T}\right)\exp\left(-\frac{mv^2}{2k_B T}\right)\right]\right] = 0$$

gives: $0 = (2v)\left[\exp\left(\frac{-mv^2}{2k_B T}\right)\right] - \left(\frac{mv^3}{k_B T}\right)\left[\exp\left(\frac{-mv^2}{2k_B T}\right)\right]$

Divide by $\exp(-mv^2/2k_B T) \rightarrow$ cancels out

gives: $0 = 2v - \frac{mv^3}{k_B T}$

$$2v = \frac{mv^3}{k_B T}$$

$$\frac{2k_B T}{m} = v^2$$

$$v = c^* = \left(\frac{2k_B T}{m}\right)^{1/2}$$ = most probable speed

H1.13

$$\langle v\rangle = \left(\frac{8kT}{\pi m}\right)^{1/2}$$

$$F(v)\,dv = 4\pi v^2\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) dv$$

$$\frac{F(\langle v\rangle)}{F(3\langle v\rangle)} = \frac{\cancel{4\pi\langle v\rangle^2}\cancel{\left(\frac{m}{2\pi k_B T}\right)^{3/2}} \exp\left(-\frac{m\langle v\rangle^2}{2k_B T}\right)}{\cancel{4\pi}\, 9\cancel{\langle v\rangle^2}\cancel{\left(\frac{m}{2\pi k_B T}\right)^{3/2}} \exp\left(-\frac{m\cdot 9\langle v\rangle^2}{2k_B T}\right)}$$

$$\frac{e^{-a}}{e^{-9a}} = e^{(-a+9a)} = e^{8a}$$

$$\text{ratio} = \frac{\exp\left(-\frac{m\langle v\rangle^2}{2k_B T}\right)}{9\exp\left(-\frac{m\langle v\rangle^2\cdot 9}{2k_B T}\right)} = \frac{1}{9}\exp\left(\frac{-m\langle v\rangle^2 + m\langle v\rangle^2\cdot 9}{2k_B T}\right)$$

$$= \frac{1}{9}\exp\left(\frac{-m\langle v\rangle^2(1-9)}{2k_B T}\right)$$

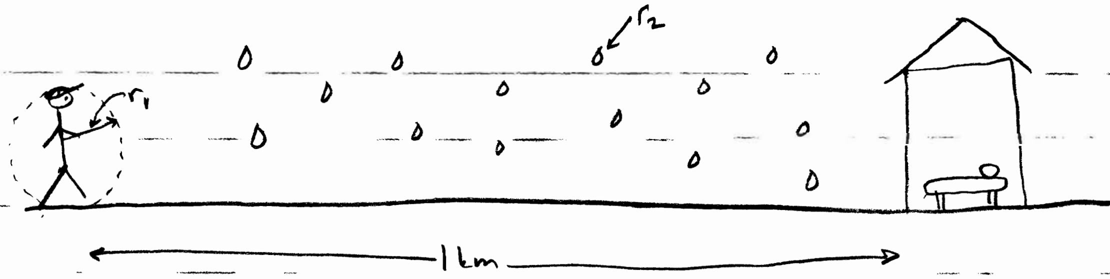
$$\text{ratio} = \frac{1}{9}\exp\left(\frac{8m\langle v\rangle^2}{2k_B T}\right)$$

but $\langle v\rangle = \left(\frac{8kT}{\pi m}\right)^{1/2}$ → plug into ratio

$$\text{ratio} = \frac{1}{9}\exp\left(\frac{8\cancel{m}\,8\cancel{k_B T}}{\pi\cancel{m}\cdot 2\cancel{k_B T}}\right)$$

$$\text{ratio} = \underbrace{\frac{1}{9}\exp\left(\frac{32}{\pi}\right)}_{\boxed{\text{independent of } T}} = \boxed{2947}$$

H1.14

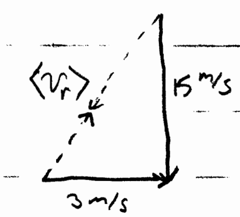


Ⓐ walk = 3 m/s   rain = 15 m/s $1000\,m\left(\frac{1\,s}{3\,m}\right) = 333\,s$

Ⓑ run = 8 m/s   rain = 15 m/s $1000\,m\left(\frac{1\,s}{8\,m}\right) = 125\,s$

$\langle v_r \rangle$ = avg. relative velocity (of rain + you)

Ⓐ



$\langle v_r \rangle = (3^2 + 15^2)^{1/2} = 15.3\,m/s$

Ⓑ $\langle v_r \rangle = (8^2 + 15^2)^{1/2} = 17\,m/s$

Z_2 = collisions of you w/ rain drops per second $= \pi b_{max}^2 \langle v_r \rangle n_2^*$ $m^2\left(\frac{m}{s}\right)m^{-3}$

n_2^* = raindrops/unit vol.

$b_{max} = r_1 + r_2 \sim r_1$ (and is equal for Ⓐ and Ⓑ)

so, $\frac{Z_{2Ⓐ}}{Z_{2Ⓑ}} = \frac{\cancel{\pi b_{max}^2}(15.3\,m/s)\cancel{n_2^*}\cdot 333\,s}{\cancel{\pi b_{max}^2}(17\,m/s)\cancel{n_2^*}\cdot 125\,s} = \frac{(15.3)(333)}{(17)(125)} = \frac{5095 \text{ collisions}}{2125 \text{ collisions}}$

so, if you walk, there will be over twice as many collisions as if you run → $\boxed{\text{running is better}}$