**Quiz 5**

1) Find the sum.

$$1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots-\frac{1}{512}$$

The first term is 1, that is $a=1$, and the common ratio $r=-\frac{1}{2}$. We need to calculate $n$ and then plug it in to the formula

$$S_n=a\left[\frac{1-r^n}{1-r}\right].$$

So $ar^{n-1}=\left(-\frac{1}{2}\right)^{n-1}=-\frac{1}{512}$, which implies that $n-1=9$, and therefore, $n=10$. Now, the required sum is

$$S_{10}=1\left[\frac{1-\left(\frac{1}{2}\right)^{10}}{1+\frac{1}{2}}\right]=\frac{341}{512}.$$

2) Find the sum of the infinite series

$$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\ldots$$

The first term $a=\frac{1}{3}$ and the common ratio $r=\frac{1}{3}$. Use the formula

$$S_\infty=\frac{a}{1-r}$$

to obtain the sum of the infinite series

$$S_\infty=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}.$$

3) Expand the expression

$$\left(x+\frac{1}{x}\right)^5$$

$$= x^5 + 5x^4\frac{1}{x} + 10x^3\frac{1}{x^2} + 10x^2\frac{1}{x^3} + 5x\frac{1}{x^4} + \frac{1}{x^5}$$

$$= x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}.$$