Name:

Quiz 4

1) Find the first five terms of the following recursively defined sequence.

$$a_n = \frac{1}{1 + 2a_{n-1}}, \quad a_1 = 0$$

$$\begin{aligned} a_1 &= 0 \\ a_2 &= \frac{1}{1+2(0)} = 1 \\ a_3 &= \frac{1}{1+2(1)} = \frac{1}{3} \\ a_4 &= \frac{1}{1+2\left(\frac{1}{3}\right)} = \frac{3}{5} \\ a_5 &= \frac{1}{1+2\left(\frac{3}{5}\right)} = \frac{5}{11} \end{aligned}$$

2) Find the sum

$$\sum_{i=1}^{5} [1 + (-1)^i].$$

$$\begin{aligned} &= [1 + (-1)] + [1 + 1] + [1 + (-1)] + [1 + 1] + [1 + (-1)] \\ &= 0 + 2 + 0 + 2 + 0 \\ &= 4 \end{aligned}$$

3) Find the sum

$$1 + 4 + 7 + 10 + \cdots + 58$$

The terms in the sum form an arithmetic sequence, with the first term $a = 1$ and the common difference $d = 3$. The expression for the nth term is $a_n = a + (n-1)d$.

First, we will determine which term is equal to 58, that is, we will solve for n in the equation $58 = a_n = 1 + (n-1)3$. It is easy to see that $n = 20$ (58 is the 20th term in the arithmetic sequence).

Now we use the formula for the sum of the first n terms to calculate the sum of the first 20 terms.

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ S_{20} &= \frac{20}{2}\left[2(1) + (20-1)3\right] \\ S_{20} &= \frac{20}{2}\left[59\right] \\ S_{20} &= (10)(59) \\ S_{20} &= 590. \end{aligned}$$

Alternatively, we could have used the other formula for the sum of the first n terms to arrive at the same answer.

$$\begin{aligned} S_n &= n\left(\frac{a + a_n}{2}\right) \\ S_{20} &= 20\left(\frac{1 + 58}{2}\right) \\ S_{20} &= 20\left(\frac{59}{2}\right) \\ S_{20} &= (10)(59) \\ S_{20} &= 590. \end{aligned}$$

Both these formulas can be derived from Gauss' method.

$$\begin{array}{rcccccccccccc} S_{20} & = & 1 & + & 4 & + & 7 & + & 10 & + & \ldots & + & 58 \\ S_{20} & = & 58 & + & 55 & + & 52 & + & 49 & + & \ldots & + & 1 \\ \hline 2\ S_{20} & = & 59 & + & 59 & + & 59 & + & 59 & + & \ldots & + & 59 \end{array}$$

So $2(S_{20}) = (20)(59)$, which implies that the sum must be 590.