

Quiz 2

1) A quadratic function is given. Express the quadratic function in standard form. Find the vertex and its intercepts. Sketch its graph, clearly indicating the vertex and the intercepts.

$$f(x) = -x^2 + 4x + 3$$

$$f(x) = -(x^2 - 4x) + 3$$

$$f(x) = -(x^2 - 4x + 4 - 4) + 3$$

$$f(x) = -(x^2 - 4x + 4) + 3 + (-4)(-1)$$

$$f(x) = -(x - 2)^2 + 7 \quad \text{Standard Form}$$

The vertex is $(2, 7)$, the y -intercept is 3, and the x -intercepts are $2 \pm \sqrt{7}$.

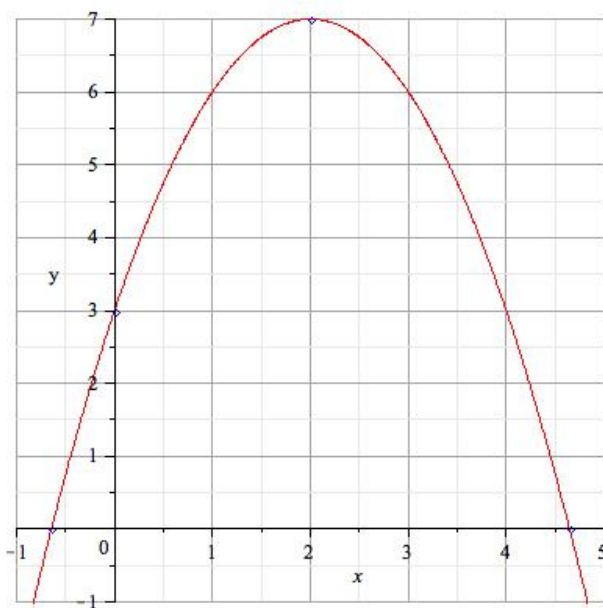


Figure 1: $f(x) = -(x - 2)^2 + 7$.

2) Find two numbers whose sum is 17 and whose product is as large as possible. Write down the quadratic function that needs to be maximized, clearly indicating what all the variables mean. Complete the square or use some appropriate formula to obtain these two numbers.

Let x and y be the two numbers with sum 17 ($x + y = 17$) and let P be the product, that is, $P = xy$. Now,

$$P = x(17 - x)$$

$$P = -x^2 + 17x$$

$$P = -\left(x^2 - 17x + \frac{17^2}{4} - \frac{17^2}{4}\right)$$

$$P = -\left(x^2 - 17x + \frac{17^2}{4}\right) + (-1)\left(-\frac{17^2}{4}\right)$$

$$P = -\left(x - \frac{17}{2}\right)^2 + \frac{17^2}{4}.$$

So the maximum occurs when $x = \frac{17}{2}$. This means that the two numbers are both equal to $\frac{17}{2}$ ($x = \frac{17}{2}$ and $y = 17 - x = \frac{17}{2}$).