FINAL

1) On $P_2(R)$ consider the inner product given by

$$\langle p, q \rangle = \int_0^1 p(x)q(x)dx.$$

Apply the Gram-Schmidt procedure to the basis $(1, x, x^2)$ to obtain an orthonormal basis of $P_2(R)$.

2) Suppose U is a subspace of a finite-dimensional vector space V. Prove that $U^\perp = \{0\}$ if and only if $U = V$.

3) Let V be a nonzero finite-dimensional vector space and let $P \in L(V)$ such that $P^2 = P$. Prove that P is an orthogonal projection if and only if P is self-adjoint.

4) Suppose V is finite-dimensional and $T \in L(V)$. Prove that T is a scalar multiple of the identity if and only if $ST = TS$ for every $S \in L(V)$. Note that T is a scalar multiple of the identity means that $T = \lambda I$ for some $\lambda \in F$, that is, $Tv = \lambda v$ for all $v \in V$.

5) Prove or disprove: there is an inner product on R^2 such that the associated norm is given by $\|(x, y)\| = |x| + |y|$ for all $(x, y) \in R^2$.

6) Let $V = \{ax^3 + bx^2 + cx : a, b, c \in R\}$. Show that V is a subspace of $P_3(R)$. Let $D \in L(V, P_2(R))$ be the differentiation map and let $T \in L(P_2(R), V)$ be the isomorphism defined by $T(x^2) = x^3, T(x) = x^2$, and $T(1) = x$. Find all eigenvalues and corresponding eigenvectors of TD. Note that $TD \in L(V)$.