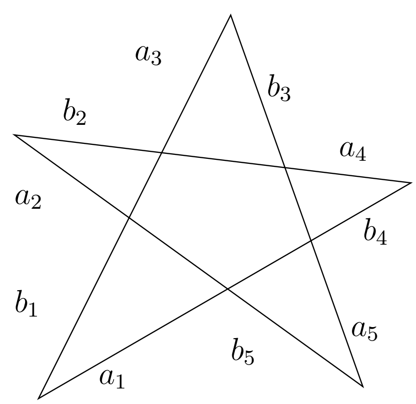
Student Mathematics Competition
Illinois Section of the
Mathematical Association of America
McKendree College, March 27, 1998

Do any four of the six problems. Put your solutions on the papers provided, beginning each problem solution on a new page. Only hand in four solutions. Entries will be graded on the basis of correctness, clarity of exposition, and elegance of solution. Enjoy the problem solving.

1. For x and y positive integers, let $f(x,y)$ denote the minimum element in $\{x, y, (\frac{1}{x} + \frac{1}{y})\}$. Determine the maximum value of $f(x,y)$.

2. Let $a = \dfrac{888\ldots89 \times 333\ldots34}{666\ldots67 \times 444\ldots45}$ and $b = 1 + \frac{1}{2}\left(\frac{1}{333\ldots34} - \frac{1}{444\ldots45}\right)$, where there are 100 digits in each of $333\ldots34$, $444\ldots45$, $666\ldots67$, and $888\ldots89$. Which is larger, a or b?

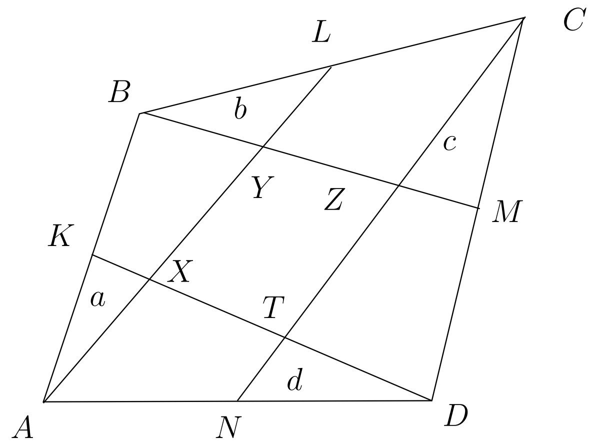
3. Five straight segments are joined to form a five-pointed star. The boundary of the star is formed by 10 line segments, whose lengths are a_i, b_i, $i = 1, 2, \ldots, 5$, as shown below. Prove that if $a_1 > b_1$, then there is a $k \in \{2, 3, 4, 5\}$ with $a_k < b_k$.



4. Find all real numbers $x_1, x_2, \ldots, x_{1998}$ such that

$$\begin{aligned} x_1x_2 &= x_1 - x_2 \\ x_2x_3 &= x_2 - x_3 \\ &\cdots \\ x_{1997}x_{1998} &= x_{1997} - x_{1998} \\ x_{1998}x_1 &= x_{1998} - x_1 \end{aligned}$$

5. Let A, B, C, D be the vertices of a convex quadrilateral; K, L, M, N the midpoints of the sides; X, Y, Z, T intersections of the lines joining vertices to midpoints of opposite sides; and a, b, c, d areas of portions of the quadrilateral as shown in the figure below. Find the area of quadrilateral $XYZT$ in terms of a, b, c, and d.



6. The graph of a non-negative, differentiable function f divides the triangle with vertices $(0, 0)$, $(0, x)$, and $(x, f(x))$ into two parts having equal areas for each positive value of x. Find an explicit expression for $f(x)$ if $f(1998) = 1998$.