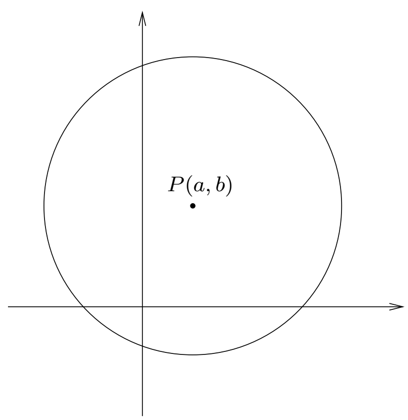
Student Mathematics Competition
Illinois Section of the
Mathematical Association of America
Rockford College, March 21, 1997

Do any four of the six problems. Put your solutions on the papers provided, beginning each problem solution on a new page. Only hand in four solutions. Entries will be graded on the basis of correctness, clarity of exposition, and elegance of solution. Enjoy the problem solving.

1. Jonathan loves candy bars. Each week, from 12:00 midnight Sunday until 12:00 midnight the following day he eats precisely 13 candy bars. Each day he eats at least one candy bar. Show if he continues this process forever there will eventually be a period of consecutive days during which he eats exactly eight candy bars.

2. Point $P(a, b)$ is in the first quadrant. A circle of radius R, center P, and containing points in each quadrant is drawn as shown below. If A_i is the area inside the circle and also the i-th quadrant, $i = 1, 2, 3, 4$, what is the value of $A_1 - A_2 + A_3 - A_4$?



3. You have a deck of cards and on each card there is a single digit between 0 and 0, inclusive. The digit on the top card equals the number of cards which have a one on them, and so forth until the digit on the last card is the number of cards with a nine on them. What are the digits, in order from top to bottom, on the cards?

4. Let f be a non-linear function which is differentiable on the interval $[a, b]$, where $a < b$. Show there is a number $c \in (a, b)$ such that
$$f'(c) > \frac{f(b) - f(a)}{b - a}.$$

5. Find all triples (x, y, z) of positive real numbers such that $x^y = z, y^z = x$, and $z^x = y$.

6. Let X be a non-empty subset of a finite group G. For $g \in G$, the set Xg is defined by $\{xg | x \in X\}$. Suppose for every pair of elements, $g_1, g_2 \in G$, either $Xg_1 = Xg_2$ or $Xg_1 \cap Xg_2 = \emptyset$. Show $X = Hw$ for some subgroup H of G and some element $w \in G$.