Student Mathematics Competition
Illinois Section of the
Mathematical Association of America
Roosevelt University - Schaumburg Campus April 2, 2004

Put your solutions on the papers provided, beginning each problem solution on a new page. Only hand in four solutions. Entries will be graded on the basis of correctness, clarity of exposition, and elegance of solution.

Enjoy the problem solving!

1. There are an odd number of terms in a certain arithmetic sequence of integers. The sum of the terms of the sequence is a power of a prime p. Prove that the number of terms of the sequence is also a power of p.

2. Find positive integers $a_1, a_2, \ldots, a_n$ such that

$$a_1 + a_2 + \cdots a_n = 100$$

and such that the product $a_1 a_2 \cdots a_n$ is maximal over all values of n and all choices of positive integers $a_1, a_2, \ldots a_n$.

3. Find all real solutions to the following equation:

$$\frac{x(x+1)}{\sqrt{x-1}} = 6x - 2 - x^2.$$

4. Ross selected five different integers and formed the ten pairwise sums. He then wrote down his results, some of which were wrong, as

$$2, 4, 5, 7, 7, 8, 10, 11, 12, 13.$$

Chris, who knew the numbers, also computed the pairwise sums and hers were all correct. Chris told Ross that the first three and the last three sums in his list were correct, but some of the others were wrong. What are the five integers? Added in proof: The three smallest correct sums are 2, 4, and 5. The three largest correct sums are 11, 12, and 13.

5. Suppose x, y, and z are the lengths of the sides of an acute triangle, no two of which are equal. Show there is a positive integer, n, such that x^n, y^n, and z^n are the sides of a triangle that is not acute.

6. Suppose $A'A$, AB, and BB' are line segments which are tangent to a circle with center O at points T_1, T_2 and T, as shown below: Let C be the intersection of $A'B$ and AB'. Show that if $A'T_1 = BT$ and $B'T_2 = AT$, then C, O, and T are collinear.