MAT 4900: History of Mathematics **Name:** ____________________________

Midterm Exam

Due, with a signed cover sheet, by Friday, October 20, 2017. (Note: electronic signatures and submissions are acceptable and encouraged, but not required.)

You are expected to work on this exam alone and to refrain from talking about the exam to anyone except the professor until the time and date when it is due. You may use your own notes and any published materials that you like. (Cite sources appropriately.)

Your signature below attests to a pledge that you have done the exam according to the above instructions. (Attach this cover page to a *printed copy* of your solutions.)

Signature:_________________________________

Solutions must be typeset in Tex.

1. (a) Derive a formula for $\sum_{i=1}^{n} i^2$ using the division of a triangular prism into three triangular pyramids. (Note: you will have to do some algebra, too!)

   (b) Prove the formula you derived.

2. (a) You may have noticed that in the addition and multiplication tables in bases $b = 7$ and $b = 12$ the entries in the lower right corner of each, for $(b-1)+(b-1)$ and $(b-1)(b-1)$, have their digits exchanged: $6+6 = 15_{(7)}$ and $6 \cdot 6 = 51_{(7)}$; $e+e = 1t_{(12)}$ and $e \cdot e = t1_{(12)}$ (where $e = 11$ and $t = 12$). Is this true in any base? If so, prove it; if not, give a counterexample.

   (b) Provide and prove a general formula for the number of digits in the numeral in base b for a number n, in terms of b and n.

   (c) Provide a general description of the regular numbers and their reciprocals as radix fractions in base 2 and in base 4.

3. (a) Derive and justify, both geometrically and analytically, a formula for the best linear approximation to $\sqrt[3]{a^3+h}$. (In other words, derive and justify the correct coefficient k in the formula $\sqrt[3]{a^3+h} = a + kh$.

   (b) Compare your result to the the approximation x_1 given by Newton's method applied to $f(x) = x^3 - (a^3+h)$ with $x_0 = a$,

4. Solve the quadratic equation $x^2-2x-2 = 0$ geometrically, using the Pythagorean method. That is, using only straightedge and compass, construct segments whose lengths are the absolute values of the roots of this equation. Clearly show your construction, and fully document and justify it. One of the roots is negative. Explain which one it is and how you know, *without* referring to the quadratic formula for the roots.

5. (a) Prove that if p and q are distinct prime numbers, then $\sqrt{p}$ and $\sqrt{q}$ are incommensurable.

   (b) Need it be true that $\sqrt[3]{p}$ and $\sqrt[3]{q}$ are incommensurable? If so, prove it. If not, give a counterexample.

   (c) What can we say in general about $\sqrt[n]{p}$ and $\sqrt[n]{q}$, where n is a positive integer greater than 1? Prove your answer is correct!