

Thurs, I. Geometric proof that $\sqrt{2} \notin \mathbb{Q}$.

9/14

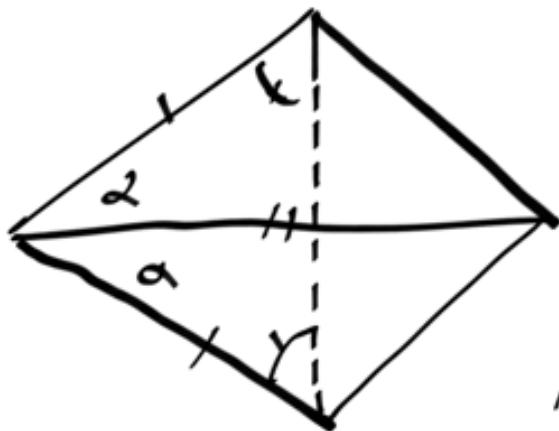
(Check up how Euclid did it.)

II. On what assumptions are our assertions based? Need EAT for greater side opp. greater k and for hyp-leg.

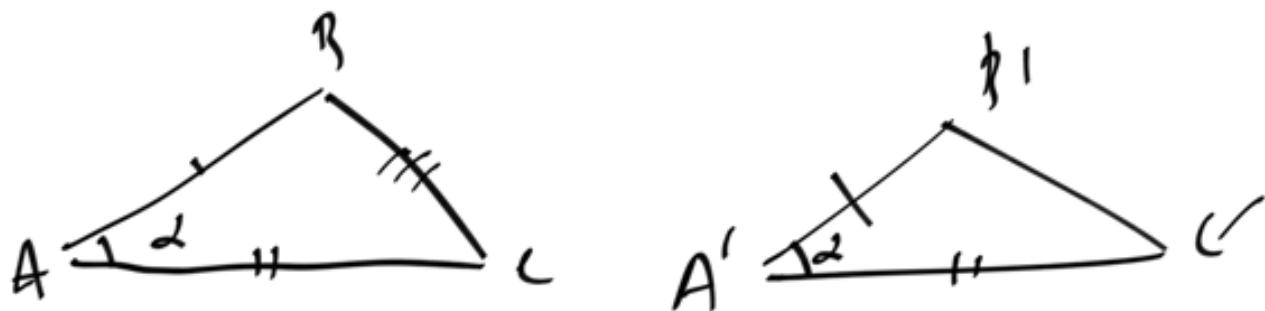
III. Two additional proofs of the irrationality of $\sqrt{2}$. All proofs ultimately rely on $\sqrt{2} = \frac{m}{n} \Leftrightarrow n\sqrt{2} = m$. (Even a commensurability argument, since $\sqrt{2}$ is comm. w/ 2 $\Leftrightarrow \exists m, n : n\sqrt{2} \in \mathbb{Z}$.) And $n\sqrt{2} = m \Leftrightarrow 2n^2 = m^2$. So we want prove there are no such m and n .

A. The geometric proof we just looked at may be rephrased as follows: If \exists pos. integers m, n s.t. $\sqrt{2} = \frac{m}{n}$, there must be a smallest scal n . Since $2 > \sqrt{2} > 1$ (what is geometric reason that $2 > \sqrt{2}^2$?), we have $1 > \sqrt{2} - 1 > 0$, so $n > m - n > 0$. Now $2(m-n)^2 = 2m^2 - 4mn + 2n^2 = m^2 - 4mn + 4n^2 > 0$, for! $= (m-2n)^2$. So setting $k = m - 2n > 0$ and $l = mn$,

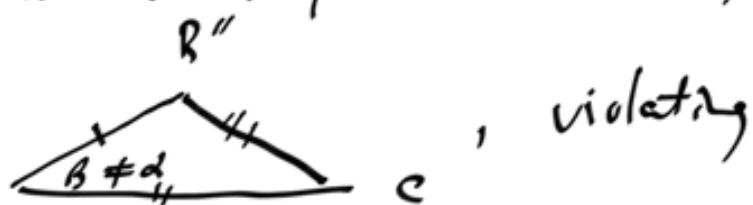
Note that base angle thm. is easily proven from SSS. But when is not.



Doesn't work.
Have to use circles:

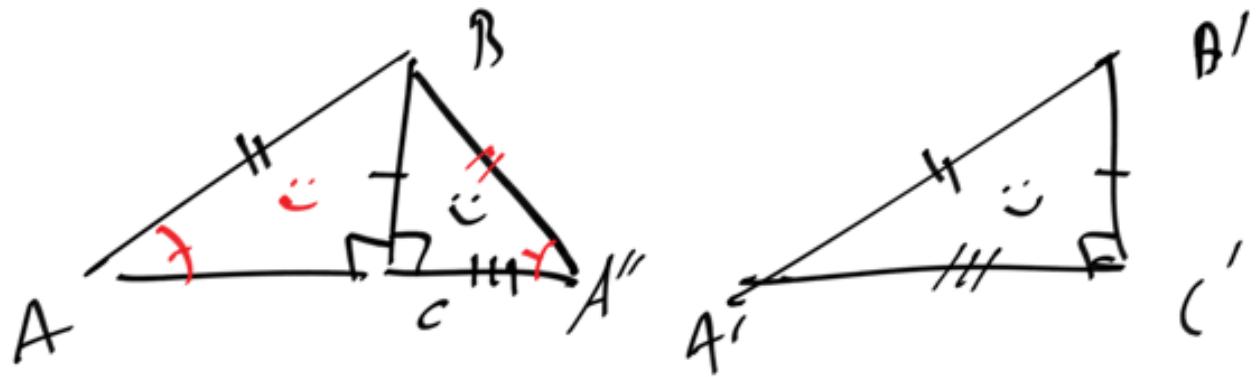


Suppose $BC \neq B'C'$. By angle subtending circles obtain

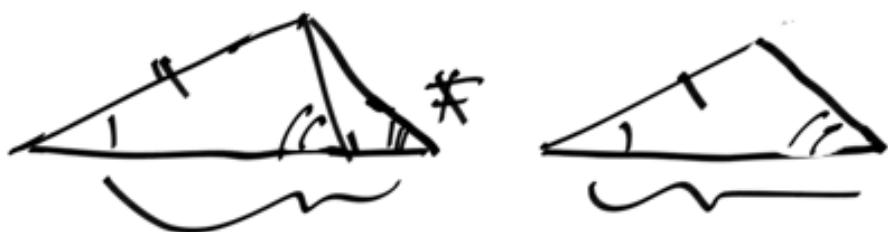


contradiction of ~~*~~ or sides of rays,

Hyp - leg :



And AAS.



Cheat Greenberg!

we have $2\lambda^2 = k^2$, \star (since $\lambda < n$)

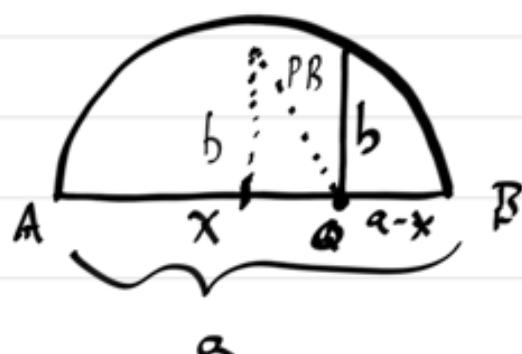
B. The preceding argument has a beautiful geometric formulation attributed to Stanley Tennenbaum, early 1950's,



$\therefore 2(a-b)^2 = (2b-a)^2$, but $a-b \in \mathbb{Z}_+$. \star'

IV. If time remains, look at square root constructions

Note:



$$\begin{aligned} x(a-x) + (PQ)^2 &= (PB)^2 \Leftrightarrow \\ x(a-x) &= PB^2 - PQ^2 \\ &= b^2 \end{aligned}$$