

- Tues 11/14
- I. Archimedes
  - II. Fund. Thm a Simple  $\Delta$
  - III. Begin discussion of constructibility  
 (1st impossibility results given by Wantzel, 1837,  
 for cube + angle)

- Thurs 11/16
- I. Recall quadratic nature of constructible field extensions; note that Omar Khayyam, Persia, 11<sup>th</sup> century, recognized that a cubic could not be solved by straightedge and compass.
  - II. Field extensions as vector spaces; some of quadratic extensions; multiplicative nature of dimension.

- III. Impossibility of duplicating cube, trisecting  $\angle A$ .  $x^3 - 2$  has no rational root, so it is irreducible over  $\mathbb{Q}$ . Ask students to show this!

B. We can show that  $\cos 20^\circ$  is not constructible; hence  $60^\circ$  cannot be trisected.

Recall:  $e^{i\theta} = \cos \theta + i \sin \theta$ , from which we can derive the addition formulas. From these

(and  $\cos 60^\circ = \frac{1}{2}$ ) we obtain that  $\cos 20^\circ$  is a root of the equation  $4x^3 - 3x - \frac{1}{2} = 0$ .

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We show this equation is solvable over  $\mathbb{Q}$  as follows: Equivalently,

$8x^3 - 6x - 1 = 0$ . Let  $y = 2x$ , obtaining  
 $y^3 - 3y - 1 = 0$ . If  $y$  is a rational sol.,  
Then  $x = \frac{y}{2}$  is a rational solution to the original  
equation. Suppose  $y = \frac{m}{n}$  is a solution in lowest  
terms.  $m^3 + 3mn^2 - n^3 = 0$ , so  $pm \Leftrightarrow pn \Rightarrow$   
 $m/n = \pm 1$ . But neither is a solution.

O. Catch up w/ trisection.

Tues. I. Some more history on field extensions.

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A. Tartaglia introduced square roots of  
negative #'s, which were more thoroughly studied  
by Bombelli. Other involved w/ the solutions  
to cubic + quartic equations were Ferrari (quartic  
in terms of cubic) and Cardano (who published  
earlier work of del Ferro/Tartaglia). All of  
this occurred in the 16<sup>th</sup> Century (late 15<sup>th</sup>  
for del Ferro). This time period saw many  
advances, including the introduction of complex #'s  
as well as imaginary #'s. Fibonacci had approxim.  
ated roots of some cubic equations in 12<sup>th</sup>-13<sup>th</sup>  
century; Bhaskara had a method that went

much sooner in 19<sup>th</sup> century India.

B. Impossibility of solving the quintic was almost completely established by Paolo Ruffini in 1799; he was the first to consider permutations of the roots as a group under multiplication. Abel filled the gap<sup>(1824)</sup>, but it was Galois who gave a thorough understanding of solvability w/ his analysis of the automorphism groups of extension fields.

C. Modern understanding of abstract field extensions. In particular, how is a "non-real" number such as  $i$  constructed.

Thus Catch up [and, if time permits, show that  $\pi$  is transcendental. This will require considerable work!!] No!

Homework - student presentations