MAT 4955/5220: Topology **Name:** ______________________________

End-of-term Exam. Due by 5 p.m. on Friday, May 1, 2020.

You are expected to work on this exam alone and to refrain from talking about the exam questions to anyone except the professor until the time and date when it is due. You are welcome to help each other learn the relevant material, but not to discuss the specific problems on the exam. You may use your own notes and any published materials that you like. (Cite sources appropriately.)

Your signature constitutes a pledge that you have done the exam according to the above instructions. (Please sign your exam at the top of whatever paper you do it on.)

Signature:______________________________

Solutions must be typeset. Electronic submission, including of this cover page, is fine.

1. (a) Prove that if $A \subset X$ is a connected subspace and x is an accumulation point of A, then $A \cup \{x\}$ is connected. (Recall that x is an accumulation point of A if every open neighborhood of x contains a point of A distinct from itself. Clearly you may assume that $x \notin A$, since the result is trivial if $x \in A$.)

   (b) Use the result of part (a) to provide an alternative proof that, if A is connected, then $\overline{A}$ is connected.

2. Let X and Y be path connected spaces each having at least two points. Let x_0 and y_0 be points of X and Y, respectively. Prove that $X \times Y - (x_0, y_0)$ is path-connected.

3. Let (X, d) be a metric space. Define the distance from a point x to a set A by $d(x, A) = \inf\{d(x, a) : a \in A\}$, as on p. 175 of Munkres.

   Define the distance between two sets A and B by $d(A, B) = \inf\{d(a, b) : a \in A \text{ and } b \in B\}$. (Note: It should be obvious that this distance does not define a metric on $\mathcal{P}(X)$.)

   (a) Prove that if $C \subseteq X$ is closed and $x \notin C$, then $d(x, C) > 0$.

   (b) Prove that if $C \subseteq X$ is compact and D is closed and disjoint from C, then $d(C, D) > 0$.

   (c) What is the case, in general, if C and D are closed and disjoint: is it necessarily the case that $d(C, D) > 0$? If so, prove it. If not, provide a counterexample.

4. (a) Prove that closed intervals in $\mathbb{Q}$, the space of rational numbers (in the usual order topology, which is the same as the subspace topology as a subspace of the real numbers), are not compact.

   (b) Prove that $\mathbb{Q}$ is not locally compact.

5. (a) Let X be a 2^{nd} countable Hausdorff space. Prove that if X is accumulation point compact, then X is sequentially compact.

   (b) Provide an example showing that the Hausdorff condition is necessary.