MAT 3530: Algebra

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You are welcome to drop in or make appointments at times other than my office hours; however, please do not knock on my door when it is closed, as I am engaged in research when in the office with the door closed. Thanks!

Text: *Elements of Modern Algebra* (7^{th} edition), by Gilbert & Gilbert (Brooks/Cole).

1. Course Content

A binary operation on a set S is a function $S \times S \to S$. An example is addition of real numbers. More generally, operations depending on a different number of inputs may be defined; for example, a unitary operation is simply a function $S \to S$. Operations between different sets can also be defined. Algebra is the study of sets on which operations are defined. Various algebraic structures, such as groups, vector spaces, and rings, are defined based on the number and properties of the operations involved.

You are undoubtedly familiar with the real number system and its subsystems, such as the rational numbers and integers. Two binary operations, addition and multiplication, are defined on the set $\mathbb{R}$ of real numbers. Both operations are associative and commutative, and multiplication distributes over addition. There are both an additive identity - 0 - and a multiplicative identity - 1, and all possible inverses exist. (As I am sure you know, 0 cannot have a multiplicative inverse.) The *inversions* $x \mapsto -x$ and $x \mapsto \frac{1}{x}$ are unitary operations. These operations and properties characterize the real number system as a *field*. The consequences and applications of this structure, such as the methods for solving equations and their application to computing when two trains will pass each other, are studied in high school algebra.

The rational number system shares all the algebraic properties of the real number system; hence, it is also a field. These two fields differ only in their analytic properties: all possible limits exist in the real number system, but not the rational number system. Another field with which you are probably familiar is the complex number system, which extends the real number system in such a way that all polynomial equations can be solved. (As a result, the complex number system is two-dimensional, losing the order relation of the real number line.)

The system of integers lacks multiplicative inverses, but all the other algebraic properties of a field apply. These operations and properties characterize the system of integers as a *commutative ring*. In fact, the integers have additional structure. Because the system of integers is a subsystem of the field of real numbers, no two non-zero integers can have zero as their product. This property characterizes the integers as a special type of commutative ring, called an *integral domain*. (The word *integral* in this label, in fact, refers to the system of integers as the archetypal example of such a structure.) Furthermore, the definition of the natural number system as the smallest inductive subsystem of the real numbers containing 1 (alternatively, 0, if one takes 0 as the smallest natural number) leads to a division algorithm for the integers, characterizing it as an even more special type of integral domain called a *Euclidean* domain. (As you would expect, this qualifier honors Euclid, who elaborated on the ramifications of this property in his *Elements*.)

You are probably also familiar with vector spaces. Only addition is defined on the set of vectors. An additive identity - $\vec{0}$ - and all additive inverses exist, making the set of vectors into a *commutative (additive) group*. Scalar multiplication is an example of an operation between two generally different sets: if F denotes the field of scalars and V denotes the group of vectors, scalar multiplication is an operation $F \times V \to V$.

The qualifier *abstract* in the title of this course simply refers to the fact that we will study the *general* consequences of the defining properties of various algebraic structures, rather than restricting our attention to specific examples. We will use examples to provide motivation and illustrations and to develop intuition.

2. Objectives and Expectations

In general, I expect students:

- To formulate precise definitions from memory and apply them correctly;
- To write rigorous proofs;
- To develop, with the aid published texts and classroom discussion, the theories of various algebraic structures and to articulate them from memory , beginning with the definition of the structure (which may be regarded as comprising the axioms of the theory of that structure), introducing additional definitions as needed, and proving the relevant theorems in logical order.

The mastery of axioms, definitions, theorems, and proofs must be based on understanding, not rote! (One way in which I will test this understanding is by asking you to work with related definitions and prove related theorems that you have not previously seen. In any case, if you attempt to learn even the standard results by rote you will surely not succeed!) Algebra is a fundamental aspect of mathematics and must be mastered!

3. Requirements

Class participation: You are expected to attend class, to read the relevant sources before class, and to work steadily on assignments. Be prepared to present your work and contribute to discussions.

Homework: Written homework assignments will be regularly assigned and graded, with comments. I encourage you to work together and discuss the course material with each other. You may hand in joint papers, although given how important it is that each student gets a lot of practice in writing definitions and proofs, I recommend that you do so sparingly and only in small groups. If you do wish to hand in a joint paper, just give credit at the top to all of the authors, who will, of course, receive the same grade. (I would much rather see a single joint paper then several essentially identical papers! If you develop your final answers together, please hand in joint work.)

Assignments must be typeset using Tex. I will provide you with a template for guidance. You are permitted two hand-written assignments during the term (in case you are unable to get to a computer or need some time to learn Tex).

Exams: There will be an early **preliminary exam**, a **mid-term exam** and a **final exam**. *The final exam will be comprehensive.* Exams will generally involve both in-class and take-home components.

In contrast to the homework assignments, *you are expected to work on take-home exams alone; no collaboration of any kind is permitted. Adherence to this rule will be strictly enforced; violations will incur serious consequences.*

Make-up exams will be given only under extraordinary circumstances or in case of serious emergency; prior permission to miss an exam must be obtained from the professor if at all possible.

4. Grading

I do not grade on a "curve". Under no circumstances will your grade directly depend on how how your fellow students do. If you do a good job of learning the material, you will receive a good grade, regardless of how well the other members of the class perform. Don't forget that the reverse is also true: if you do a poor job of learning the material, you will receive a poor grade, regardless of how poorly everyone else does.

Most questions on assignments and exams will require proofs or other essay responses. For such questions, I will assign letter (rather than numerical) grades, based on specified objectives and standards. These letter grades will be converted to the standard $0-4$ scale, as will percentage scores (using the standard scale: $90\% \leq$ A $\leq 100\%$, $80\% \leq$ B $< 90\%$), etc.), and a weighted average will then be used to compute your grade for the course.

Grades correspond to my judgement of quality as follows:

- A Excellent. The work exhibits mastery of nearly all important ideas, including those which are subtle or difficult, much insight and originality, highly coherent organization and fine expository style. Errors and omissions, if any, are minor.
- B Good. The work exhibits substantial understanding of most important ideas, including some which are subtle or difficult, some insight and originality, coherent organization and correct usage, grammer and spelling. There are some substantive errors or omissions.
- C Fair. The work exhibits basic understanding of many fundamental ideas, although not those which are subtle or difficult, and demonstrates some coherence. The presentation is readable, with at most minor errors of usage, grammer or spelling. There are many substantive errors or omissions.
- D Poor. The work exhibits some understanding of a few fundamental ideas, but not those which are subtle or difficult, and it fails to demonstrate coherence. Usage, grammar and spelling are mostly correct. There are a great many subtantive errors or omissions.
- F No credit. The work exhibits too few of the positive qualities noted above to be worthy of credit.

Each requirement will count toward your final grade as follows (possibly subject to slight modification):

Homework:	30%
Preliminary Exam:	10%
Mid-term exam:	30%
Final Exam:	30%

Complete honesty on assignments and exams is expected of all students. All sources must be appropriately cited and acknowledged.