MAT 3530, Algebra: Preliminary Exam **Name:** ______________________________

February 20, 2012

No calculators, notes, or books are allowed. (Can't imagine what you would do with a calculator in abstract algebra, in any case!) You will need writing implements, a compass, and a straightedge.

Each numbered question is worth 20 points; any lettered parts have the same value.

1. Compute the following products in S_3, the group of permutations of the set $\{1, 2, 3\}$. Write your answer as a single cycle; for example, $(12)(23) = (123)$. Use e to denote the identity element.

   (a) $e(123) = \underline{(123)}$

   (b) $(123)^2 = \underline{(132)}$

   (c) $(12)(13) = \underline{(132)}$

   (d) $(12)^2 = \underline{e}$

2. Compute each of the following products in S_4, the group of permutations of the set $\{1, 2, 3, 4\}$. Write your answer as a product of disjoint cycles (possibly a single cycle); for example, $(123)(234) = (12)(34)$, and $(12)(14) = (142)$. As always, e denotes the identity element.

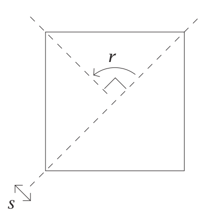
   (a) $(12)(13) = \underline{(132)}$

   (b) $(12)(13)(14) = \underline{(1432) = (4321)}$

   (c) $(1234)^3 = \underline{(1432) = (4321)}$

   (d) $(1234)^4 = \underline{e}$

3. Compute each of the following products in the dihedral group D_4, the group of symmetries of a square. Write your answer in the form $r^i s^j$, where s and r are the reflection and rotation indicated and i and j are the minimal positive powers, leaving out any factors equal to the identity (unless the entire product is the identity, in which case write e); for example, $sr^3 = rs$, $s^2 r = r$, $s^3 = s$, and $r^4 = e$.



   (a) $srs = \underline{r^3}$

   (b) $rsr = \underline{s}$

   (c) $rs^2 r = \underline{r^2}$

   (d) $r^5 = \underline{r}$

-over-

4. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if both f and g are injective, then $g \circ f$ is injective.

Proof. Assume both f and g are injective and that $g \circ f(a) = g \circ f(a')$. By definition of composition, $g \circ f(a) = g(f(a))$ and $g \circ f(a') = g(f(a'))$; hence, $g(f(a)) = g(f(a'))$. Since g is injective, $f(a) = f(a')$, and since f is injective, $a = a'$. Thus, for any $a, a' \in A$, $g \circ f(a) = g \circ f(a') \Rightarrow a = a'$, which by definition shows that $g \circ f$ is injective. □

5. Let $\rho_{O,\theta}$ be the counterclockwise rotation in the plane by angle θ with center O. Using straightedge and compass *only, precisely* construct $\rho_{O,\theta}(P)$, where O, θ, and P are as given below.