MAT 3530: Algebra **Name:** ______________________________

Preliminary Exam: take-home portion. Due at the beginning of class on Wednesday, February 22. *Solutions must be typed in Latex.*

You are expected to work on this exam alone and to refrain from talking about the exam to anyone except the professor until the time and date when it is due. You may use your own notes and any published materials that you like. Published sources (whether hard-copy or on the Web) must be appropriately cited!

Your signature below attests to a pledge that you have done the exam according to the above instructions. (Please attach this cover page to your solutions.)

Signature:______________________________

1. (a) Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is injective, then f is injective.

   (b) Is the converse true? If so, prove it; if not, give a counterexample.

2. Let $\tau_{\vec{v}}$ denote translation by the vector $\vec{v}$, and let ϕ_l denote reflection in the line l.

   (a) In the general case that $\vec{v}$ and l are not perpendicular, prove that $\tau_{\vec{v}} \circ \phi_l$ and $\phi_l \circ \tau_{\vec{v}}$ are both glide reflections. In each case, identify the glide vector and line of reflection.

   (b) In the special case that $\vec{v}$ and l are perpendicular, prove that $\tau_{\vec{v}} \circ \phi_l$ and $\phi_l \circ \tau_{\vec{v}}$ are both reflections. In each case, identify the line of reflection.

   Hints: For both parts, arrange $\vec{v}$ so that l passes through its midpoint. Identify the line of reflection (which will be fixed by the composition) first. To see the glide vector in the general case, consider a right triangle with $\vec{v}$ as hypotenuse and one side perpendicular to l. (The special case may be viewed as simply having glide vector $\vec{0}$.)