

**MAT 3530, Algebra: Preliminary Exam**  
**February 20, 2012**

**Name:** \_\_\_\_\_

*No calculators, notes, or books are allowed. (Can't imagine what you would do with a calculator in abstract algebra, in any case!) You will need writing implements, a compass, and a straightedge.*

*Each numbered question is worth 20 points; any lettered parts have the same value.*

1. Compute the following products in  $S_3$ , the group of permutations of the set  $\{1, 2, 3\}$ . Write your answer as a single cycle; for example,  $(12)(23) = (123)$ . Use  $e$  to denote the identity element.

(a)  $e(123) =$  \_\_\_\_\_

(b)  $(123)^2 =$  \_\_\_\_\_

(c)  $(12)(13) =$  \_\_\_\_\_

(d)  $(12)^2 =$  \_\_\_\_\_

2. Compute each of the following products in  $S_4$ , the group of permutations of the set  $\{1, 2, 3, 4\}$ . Write your answer as a product of disjoint cycles (possibly a single cycle); for example,  $(123)(234) = (12)(34)$ , and  $(12)(14) = (142)$ . As always,  $e$  denotes the identity element.

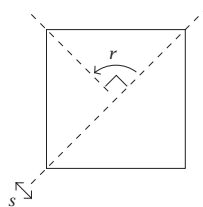
(a)  $(12)(13) =$  \_\_\_\_\_

(b)  $(12)(13)(14) =$  \_\_\_\_\_

(c)  $(1234)^3 =$  \_\_\_\_\_

(d)  $(1234)^4 =$  \_\_\_\_\_

3. Compute each of the following products in the dihedral group  $D_4$ , the group of symmetries of a square. Write your answer in the form  $r^i s^j$ , where  $s$  and  $r$  are the reflection and rotation indicated and  $i$  and  $j$  are the minimal positive powers, leaving out any factors equal to the identity (unless the entire product is the identity, in which case write  $e$ ); for example,  $sr^3 = rs$ ,  $s^2 r = r$ ,  $s^3 = s$ , and  $r^4 = e$ .



(a)  $sr s =$  \_\_\_\_\_

(b)  $r s r =$  \_\_\_\_\_

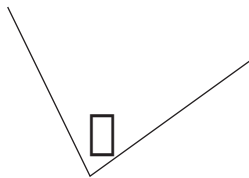
(c)  $r s^2 r =$  \_\_\_\_\_

(d)  $r^5 =$  \_\_\_\_\_

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4. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions. Prove that if both  $f$  and  $g$  are injective, then  $g \circ f$  is injective.

5. Let  $\rho_{O,\theta}$  be the counterclockwise rotation in the plane by angle  $\theta$  with center  $O$ . Using straightedge and compass *only, precisely* construct  $\rho_{O,\theta}(P)$ , where  $O$ ,  $\theta$ , and  $P$  are as given below.



$\cdot$   
 $P$

$\cdot$   
 $O$