

MAT 3530: Assignment 4

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January 29, 2012

Due at the beginning of class on Friday, February 10.

1 More Products in $\text{Isom}(\mathbb{R}^2)$

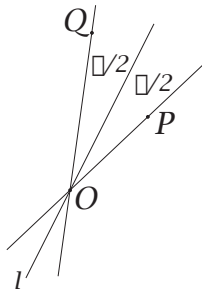
In the previous assignment, you showed that the product of two reflections is a rotation or translation, and you identified the angle and center of rotation or the vector of translation, respectively. Recall that the computation broke into cases based on whether or not the lines of reflection were distinct and non-parallel. You also showed that the product of two translations is a translation and identified the vector of translation. Finally, you showed that the product of two rotations is a rotation or translation, and you identified the the angle and center of rotation or the vector of translation, respectively. The computation broke into cases based on whether or not the sum of the two angles differed from 2π .

In the process of doing these computations, you learned some useful techniques: You can tell an orientation-preserving isometry is a rotation and identify its center if you can find a fixed point. You can identify the angle of rotation from the angle between any directed line and its image. If a directed line is taken to a parallel directed line (with the same direction of orientation), then there is no rotation, and the isometry (if orientation-preserving) must be a translation. Also note that any orientation-reversing isometry fixes some line as a set; the key to identifying the isometry is to find this line. If it is fixed pointwise, then the isometry is reflection in that line; otherwise, the vector from a point on the line to its image tells you the glide.

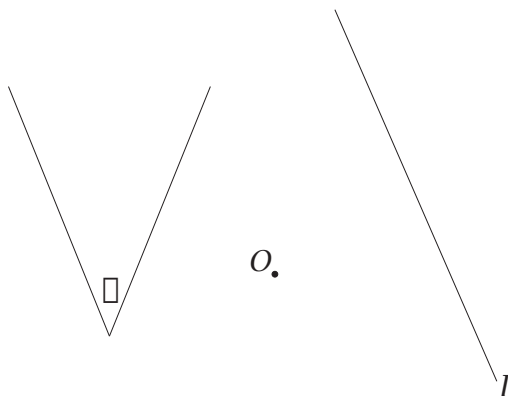
Now we will consider the other combinations of isometries. The case of a reflection composed with a rotation is probably the most difficult, so we'll handle it first. The case of a rotation composed with a translation is somewhat easier, at least in my opinion. The case of a reflection composed with a translation, which is the easiest in my view, will be left for the preliminary exam.

- (a) The composition of a rotation and a reflection.

Case 1: The center of rotation lies on the line of reflection, as in the diagram below; in this case, note that the center is fixed by both the reflection and the rotation. Lines \overrightarrow{OP} and \overrightarrow{OQ} have been constructed so that $\angle POQ = \theta$ and l bisects $\angle POQ$. (Make sure you now how to do this for an arbitrary angle with straightedge and compass!) Prove that $\phi_l \circ \rho_{\theta, O} = \phi_{\overrightarrow{OP}}$ and $\rho_{\theta, O} \circ \phi_l = \phi_{\overrightarrow{OQ}}$. Construct a diagram to illustrate your proof.



(b) **Case 2:** The center of rotation does not lie on the line of reflection, as in the diagram below.

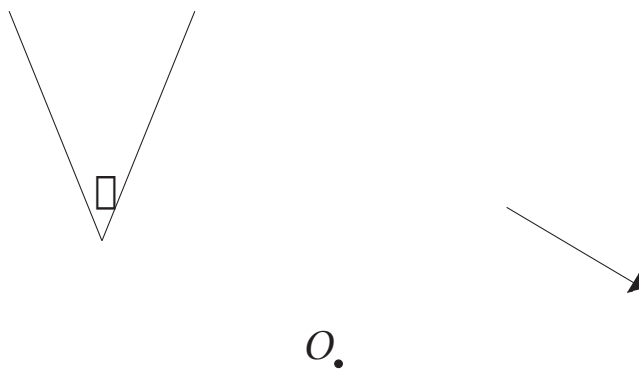


In this case, let M be the foot of the perpendicular from O to l . Let L and N lie on l such that $LM \cong NM$ and $\angle LON = \theta$. Let P be the foot of the perpendicular from M to OL and Q be the foot of the perpendicular from M to ON . Construct all of these points and:

- i. Prove $\phi_l \circ \rho_{\theta, O}$ is a glide reflection in line \overleftrightarrow{PM} . Construct and identify the glide vector.
- ii. Prove $\rho_{\theta, O} \circ \phi_l$ is a glide reflection in line \overleftrightarrow{QM} . Construct and identify the glide vector.

2. The composition of a rotation and a translation. Given center O , angle θ , and vector \vec{v} below, construct points P and Q such that:

- $\overrightarrow{QP} = \vec{v}$,
- $OP \cong OQ$, and
- $\angle POQ = \theta$.



Prove that $\tau_{\vec{v}} \circ \rho_{\theta, O} = \rho_{\theta, P}$ and $\rho_{\theta, O} \circ \tau_{\vec{v}} = \rho_{\theta, Q}$.

2 The Integers

Do the following exercises from our text. (This numbering provides a template for your solutions.)

2.3 Divisibility

17.

18.

19.

20.

21.

22.

23.

24.

25.

26.

31.