MAT 3271: Geometry

Text: *Euclidean and Non-Euclidean Geometries*, by Marvin Jay Greenberg
Required Tools: compass, straightedge

Professor: Charles Delman	Professor's Office: M3216
Email: cidelman@eiu.edu	Office Hours: MWTR@4p.m.
Office Phone: 581-6274	Home Phone: 348-7786 (before 9 p.m., please)

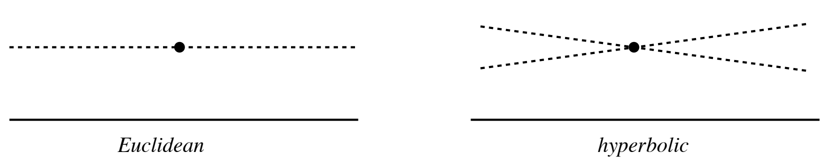
1. Course Content

Euclid and the origins of axiomatic geometry; constructions with straightedge and compass; the axiomatic method: undefined terms, definitions, axioms, theorems, logic and proof; incidence geometry; models of axiomatic systems; Hilbert's axioms; neutral geometry; modern Euclidean geometry; introduction to hyperbolic geometry, it's historical development, and its impact; relationships between geometry and algebra. (Note: this course is the first half of a two course sequence, continued by MAT 3272 .)

2. Objectives

- The student will independently write clear, logically sound definitions and proofs.
- The student will be able to discern the errors of reasoning in an incorrect proof and the unstated assumptions made in a non-rigorous argument.
- The student will present ideas to the class in a clear and organized fashion, listen attentively to the presentations of others and, in a polite, respectful and constructive manner, point out errors, raise questions, and offer suggestions for correction or improvement.
- The student will be able to verify that the axioms of a system hold in a model of that system, construct models of simple axiomatic systems, and demonstrate an isomorphism between two isomorphic models.
- Without reference to external sources, the student will be able:
  - to define the major concepts of geometry which have been covered and explain their purpose;
  - to state and prove the major theorems of geometry which have been covered and discuss their consequences.
- Without reference to external sources, the student will be able to prove statements whose proofs have not previously encountered.

The purpose of this course is to study geometry critically, rigorously, and from a historical perspective. We will begin with the geometry presented by Euclid (the kind you learned in high school), examinining its axioms and exposing Euclid's hidden assumptions. We will make these assumptions explicit and put Euclidean geometry on a modern rigorous foundation using the axiom system developed by Hilbert. Then we will discuss the independence from the other axioms of the postulate on parallel lines and begin the study of hyperbolic geometry, which is the system which results if the Euclidean Parallel Postulate is replaced by its negation.



The development of modern geometry centers on the long controversy surrounding Euclid's fifth postulate, which is equivalent to the statement that, given any line l and any point P not on l, there is exactly one line through P which is parallel to l. We will call this the Euclidean Parallel Postulate. Although the Euclidean Parallel Postulate may seem obvious to you, we will see that there is absolutely no more validity in assuming it than in assuming its opposite, namely that there exists a line l and a point P not on l such that at least two distinct lines through P are parallel to l. We will call this opposite assumption the Hyperbolic Parallel Postulate, and the geometric system which results from assuming it is called hyperbolic geometry. By studying models of hyperbolic geometry we will develop a concrete understanding of its existence. Hyperbolic geometry has some surprising properties; for example, all similar triangles are congruent - there is no *scaling*! It also has revolutionary applications.

I hope that, in addition to the specific objectives stated above, you will come to appreciate that geometry, and mathematics as a whole, is a developing science rather than a rigid set of rules handed down from the past, and that its history has not been without heated controversies. I also hope you will develop an awareness of the aesthetic and mysterious qualities of mathematics and enjoy the beautiful and often surprising discoveries which have made the history of geometry so exciting.

3. Requirements

Class participation: You are expected to be in class every day and to be prepared to present your progress toward a solution of any of the assigned problems. I will seek to engage every member of the class in discussion. Effort and attentiveness are what matters; mistakes are o.k.!

Homework: Written homework problems will be regularly assigned and graded (with comments).

In-term Exams: There will be three exams during the class term (in addition to the final), which may be partly given as take-homes. Make-up exams will be given only under extraordinary circumstances or in case of serious emergency; prior permission to miss an exam must be obtained from the professor if at all possible.

Final exam: The final exam will be comprehensive. It may be partly given as a take-home.

4. Grading

I do not grade on a "curve". Under no circumstances will your grade directly depend on how how your fellow students do. If you do a good job of learning the material, you will receive a good grade, regardless of how well the other members of the class perform. Don't forget that the reverse is also true: if you do a poor job of learning the material, you will receive a poor grade, regardless of how poorly everyone else does.

I will assign letter (rather than numerical) grades, based on the objectives stated above and standards clarified in class. Grades correspond to my judgement of quality as follows:

- A Excellent. The work exhibits mastery of nearly all important ideas, including those which are subtle or difficult, much insight and originality, highly coherent organization and fine expository style. Errors and omissions, if any, are minor.
- B Good. The work exhibits substantial understanding of most important ideas, including some which are subtle or difficult, some insight and originality, coherent organization and correct usage, grammer and spelling. There are some substantive errors or omissions.
- C Fair. The work exhibits basic understanding of many fundamental ideas, although not those which are subtle or difficult, and demonstrates some coherence. The presentation is readable, with at most minor errors of usage, grammer or spelling. There are many substantive errors or omissions.
- D Poor. The work exhibits some understanding of a few fundamental ideas, but not those which are subtle or difficult, and it fails to demonstrate coherence. Usage, grammar and spelling are mostly correct. There are a great many subtantive errors or omissions.
- F No credit. The work exhibits too few of the positive qualities noted above to be worthy of credit.

Each requirement will count toward your final grade according to the scheme below (possibly subject to slight modification):

Homework and class participation:	40%
In-term Exams:	30%
Final Exam:	30%

Complete honesty on assignments and exams is expected of all students.