MAT 3271: Geometry **Name:** ______________________________

Third Exam; Due by 2 p.m. on Wednesday, December 2, 2015.

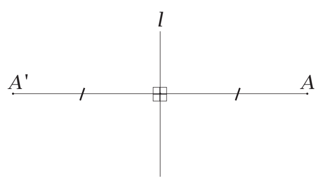
You are expected to work on this exam alone and to refrain from talking about the exam to anyone except the professor until the time and date when it is due. You may use your own notes and any published materials that you like. Be sure to cite any published sources that you use.

Your signature below attests to a pledge that you have done the exam according to the above instructions. You must attach a signed cover page to your solutions.

Signature:______________________________

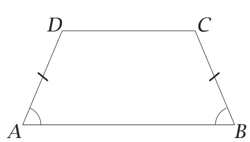
1. Given $\triangle ABC$, let D be the midpoint of BC, and let E be the point on the ray opposite $\overrightarrow{DA}$ such that $DE \cong DA$. Prove that $\angle AEC \cong \angle EAB$.

2. Given a line l and a point A not lying on l, define the *reflection in* l of point A as follows: let Q be the foot of the perpendicular from A to l. (It is an immediate consequence of the Alternate Interior Angle Theorem, which we will discuss on Friday, November 20, that the perpendicular to l through A is unique; the point at which this perpendicular intersects l is called its *foot*. The reflection in l of point A is the point A' on line $\overleftrightarrow{AQ}$ on the ray opposite $\overrightarrow{QA}$ such that $QA' \cong QA$. (See the figure below.



Now consider a line l and two points, A and B, on the same side of l. Let A' and B' be their reflections in l. Prove that $A'B' \cong AB$. (Note that there are two cases: point B may lie on line $\overleftrightarrow{AA'}$ or it may not.)

3. Prove that if, in quadrilateral $\square ABCD$, $\angle A \cong \angle B$ and $AD \cong BC$, then $\angle C \cong \angle D$.



4. Archimedes' Axiom for the real number system states: For any positive real number ζ, there is a natural number n such that $\zeta < n$. (Recall that the natural numbers are $1, 2, 3, \ldots$.) Prove that Archimedes' Axiom implies that for any positive real number ϵ, there is a natural number n such that $\frac{1}{n} < \epsilon$. You may use the algebraic and order properties of the real number system without proof.