MAT 3271: Geometry **Name:** ______________________________

Second Exam; Due by 2 p.m. on Monday, November 9, 2015.

You are expected to work on this exam alone and to refrain from talking about the exam to anyone except the professor until the time and date when it is due. You may use your own notes and any published materials that you like. Be sure to cite any published sources that you use.

Your signature below attests to a pledge that you have done the exam according to the above instructions. You must attach a signed cover page to your solutions.

Signature: ______________________________

1. In the real projective plane, compute the points of intersection of the lines determined by the following pairs of equations:

   (a)
   $$x + y + z = 0$$
   $$z = 0$$

   (b)
   $$x + y + z = 0$$
   $$x + y + 2z = 0$$

   (c)
   $$x + 2y + z = 0$$
   $$2x + y + z = 0$$

   In the real projective plane, compute an equation for the line through the following pair of points:

   (d) $[1, 0, 1]$ and $[0, 1, 0]$

   (You should be able to do all parts of this problem by inspection, without much algebra or computation.)

2. Let $\mathcal{A}$ be an affine plane. Consider the dual interpretation: points are the lines of $\mathcal{A}$ and lines are the points of $\mathcal{A}$; a point and line are incident if they are incident in $\mathcal{A}$.

   Show that Incidence Axiom 1 fails in this interpretation, but that Incidence Axioms 2 and 3 hold.

3. Prove the analog of Proposition 3.3 for rays: If $r * s * t$ and $r * t * u$, then $s * t * u$ and $r * s * u$. (See problem 15 of Chapter 2 for the notation for betweenness of coterminal rays. Hint: Use the Crossbar Theorem, Proposition 3.7, and Proposition 3.3.)

4. (a) Is it true that, if three rays are coterminal, then one must be between the other two? If it is, prove it. If it is not, give a counterexample. Describe your counterexample precisely and prove that the statement fails to hold for it. (A picture without a proof will earn a small amount of partial credit.)

   (b) Is it true that, given three coterminal rays, *at most* one can be between the other two? If so, prove it. If not, give a counterexample. [Hint: Assume one ray is between the other two and use Proposition 3.7.]