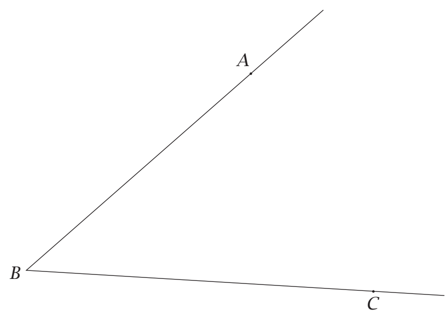
MAT 3271: Exam 1 **Name:** ______________________________
October 6, 2010.

Constructions, Definitions, Incidence Theorems, & Models

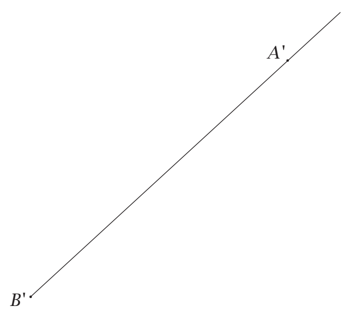
Each question is worth 10 points.

Constructions. Construct the following using only straightedge and compass. Show all construction marks clearly.

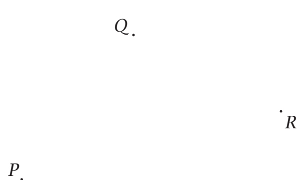
1. The bisector of $\angle ABC$.



2. A ray $\overrightarrow{A'C'}$ such that $\angle A'B'C' \cong \angle ABC$. (Use whichever side of line $\overleftrightarrow{A'B'}$ is most convenient.)



3. The circle passing through points P, Q, and R.



Definitions. Define the following terms.

4. Given points A and B, define *segment* AB.

5. Given points O and P, define the *circle with center* O *and radius* OP.

6. Define what it means for a pair of angles to be *supplementary.* You may assume that the following terms have been defined: angle, side of an angle, ray, opposite ray.

7. Define what it means for an angle to be a *right* angle.

Propositions. Give complete, well-organized proofs of the following propositions.

8. **Proposition 2.1** If l and m are distinct lines that are not parallel,then l and m have a unique point in common.

9. **Proposition 2.2** There exist three distinct lines that are not concurrent.

Models.

1. Consider the following interpretation of incidence geometry: Let A, B, C, and D be the distinct sets $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$, and $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, respectively. The points of the model are A, B, C, and D. The lines of the model are the sets $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{B, C\}$, $\{B, D\}$, $\{C, D\}$. Point P and line l are incident if (and only if) $P \in l$.

   Verify, with clear and complete proofs, that this interpretation is an affine plane. That is, verify that it satisfies the three axioms of incidence geometry and also the Euclidean Parallel Postulate.