

MAT 2800: Foundations of Mathematics

Professor: Charles Delman

Email: cidelman@eiu.edu (I do not have an office phone; please use email!)

Professor's Office: OM 3216

Office Hours: MWR 2-3:50 p.m.

Text: *Patterns of Structure and Argument*, by Charles Delman. (Copies will be distributed and are available on the course Web site.) This book will be the main text for the course. I have written and revised it specifically for this course over many years of teaching it, based on the needs of students.

How to Prove It: A Structured Approach, by Daniel J. Velleman, Cambridge University Press. This book, available from textbook rental, is quite good and should prove to be a useful supplementary reference.

1. COURSE CONTENT

This is a course in the methods, meaning, and fundamental structures of theoretical mathematics. Topics include logic and proof, the nature of mathematical theory (axioms, definitions, theorems), the natural, rational, real, and (if time permits) complex number systems, induction, recursion, and an introduction to axiomatic set theory and the rigorous definition of functions and relations. These concepts are used throughout mathematics and will provide a foundation for your future study, hence the course title.

2. OBJECTIVES AND EXPECTATIONS

In general, I expect students to learn to:

- Formulate and apply precise definitions;
- Write rigorous proofs;
- Develop a theory, beginning with clearly stated axioms and introducing precise definitions and clearly stated theorems in logical order;
- Articulate the relationships among mathematical concepts, assumptions, and results.

In addition to the general abilities outlined above, which will be developed in the course of study, I expect students to know all of the definitions, axioms, and theorems that are covered and be able to prove these theorems correctly without reference to notes, books, or other materials. I expect proofs to be based on understanding, not memorization! (I will test this understanding by asking you to prove modifications of these theorems or related theorems that you have not studied before.) It is necessary to know these basic theories so well because they are so fundamental!

3. REQUIREMENTS

Class participation: You are expected to attend class, to read the relevant sources before class, and to work steadily on assignments. Be prepared to present your work and contribute to discussions.

Homework: Written homework assignments will be regularly assigned and graded, with comments. Doing the assignments is the primary tool for learning the material. The process outlined below is meant to ensure that you take the assignments seriously and obtain the maximum benefit from doing them.

You will be expected to turn in a draft of your solutions to each assignment, which will be returned with brief comments and symbolic markings indicating the location of common mistakes (see *Symbol Chart*) in a selection (possibly all) of the problems. You are expected to make this initial draft as good as possible. A final draft of your solutions will be due several days later. No assignment will be accepted for grading unless the initial draft has been turned in on time, and I reserve the right to refuse to accept your final draft if your initial draft shows insufficient care and effort.

An exclamation point (“!”) at the top of your initial draft indicates that you *must* see me to discuss the assignment before your final draft will be accepted. (An exclamation point does not necessarily mean you have not put in enough effort; it just means you have made errors serious enough to indicate the need for one-on-one assistance.) *As noted above, the purpose of the process described here is that you will learn from doing the homework!*

I encourage you to work together and discuss the course material with each other. You may hand in joint papers with up to three authors, although given how important it is that each student gets a lot of practice in writing definitions and proofs, I recommend that you do so very sparingly. If you choose to write a joint paper, I strongly advise that you do not divide up the problems, but rather work on and discuss them all until you are satisfied with the solutions as you write them up. Give credit at the top to all of the authors, who will, of course, receive the same grade. (It is up to you to ensure that everyone contributes fairly. If your paper receives an exclamation point, you are all expected to come see me together.)

I encourage you to submit assignments electronically to the dropbox provided for this purpose. Alternatively, you may submit a hard copy at the beginning of class or in my faculty mailbox before class. Late assignments will not generally be accepted, although exceptions can be made when extenuating circumstances warrant them..

All assignments must be typeset using Tex. (I will provide a template and initial introduction.) The purpose of using Tex is three-fold: it is easier to revise your work, because you can cut, paste, and type without starting over; your work is easier to read; Tex is the standard for mathematical printing, so you should know it, and it is not hard to learn (especially these days when you can Google anything).

Multiple pages must be stapled. (No paper clips, folded corners, torn edges, etc. Origami is wonderful, but not for attaching pages to each other.)

Papers that do not meet these basic formal requirements will not be accepted.

Exams: There will be three exams during before final exam period and a final exam. The first exam will be given in class near the beginning of the term, closed book and closed notes. It's purpose is to assess whether or not you have learned the basic definitions and concepts. The second exam (*mid-term exam*) will be given near the middle of the term and cover all material since the beginning of the course. The third exam (*end-of-term exam*) will be given near the end of the term and cover all material since the midterm. The final exam will cover all the material in the course.

The midterm, end-of-term, and final exams may include take-home portions. In contrast to the homework assignments, you are expected to work on take-home exams alone; no collaboration of any kind is permitted.

Make-up exams will be given only under extraordinary circumstances or in case of serious emergency; prior permission to miss an exam must be obtained from the professor if at all possible.

4. GRADING

I do not grade on a "curve". Under no circumstances will your grade directly depend on how your fellow students do. If you do a good job of learning the material, you will receive a good grade, regardless of how well the other members of the class perform. Don't forget that the reverse is also true: if you do a poor job of learning the material, you will receive a poor grade, regardless of how poorly everyone else does.

Most problems on assignments and exams will require proofs or other essay responses. For such problems I will assign letter (rather than numerical) grades, based on specified objectives and standards. These letter grades will be converted to the standard 0 – 4 scale, as will numerical scores ($100\% \equiv 4.5, 90\% \equiv 3.5, \dots, 60\% \equiv 1.5, \leq 55\% \equiv 0$), and a weighted average will then be used to compute your final grade for the course.

Letter grades correspond to my judgement of quality as follows:

- A Excellent. The work exhibits mastery of nearly all important ideas, including those which are subtle or difficult, much insight and originality, highly coherent organization and fine expository style. Errors and omissions, if any, are minor.
- B Good. The work exhibits substantial understanding of most important ideas, including some which are subtle or difficult, some insight and originality, coherent organization and correct usage, grammar and spelling. There are some substantive errors or omissions.
- C Fair. The work exhibits basic understanding of many fundamental ideas, although not those which are subtle or difficult, and demonstrates some coherence. The presentation is readable, with at most minor errors of usage, grammar or spelling. There are many substantive errors or omissions.
- D Poor. The work exhibits some understanding of a few fundamental ideas, but not those which are subtle or difficult, and it fails to demonstrate coherence. Usage, grammar and spelling are mostly correct. There are a great many substantive errors or omissions.
- F No credit. The work exhibits too few of the positive qualities noted above to be worthy of credit.

Each requirement will count toward your final grade as follows (possibly subject to slight modification):

$$\text{Homework: } 30\%; \quad \text{Exams: } 3 \times 15\% = 45\%; \quad \text{Final Exam: } 25\%.$$

Complete honesty on assignments and exams is expected of all students. All sources must be appropriately cited and acknowledged.