Informal Argumentation: Practice to Cultivate Intuitive Logic

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Purpose

- Mathematical knowledge is established by logical deduction. The process of defining concepts and conceiving directions of inquiry involves intuition and vision, but in the end one must be able to prove ones results and to understand and verify the proofs of others.
- A basic deductive argument consists of a series of assertions, called premises, followed by the assertion of one or more conclusions. "Therefore" or some equivalent expression introduces the conclusions. Several basic arguments may be combined in sequence, with the conclusions of previous arguments used as premises in later ones.
- An argument is *valid* if its conclusions follow logically from the premises, meaning that they must be true under the condition that the premises are true.

Purpose, Continued

- Whether or not the premises actually are true is not considered in judging the validity of an argument.
- In order to be established as a *theorem* of mathematics, an assertion that is not part of a definition or axiom must be reached as a conclusion of a valid argument, using as premises only definitions, axioms, or theorems that have been previously established in this way.
- *Proof* is the process of establishing a theorem. (To carry out this process is to *prove* the theorem.)
- Often the argument used in the proof gives as much insight as the statement of the theorem itself.
- You can think of a mathematical theory, such as algebra, Euclidean geometry, or number theory as one big argument, with theorems noted along the way as they are proven!

Purpose, Continued

- Formal mathematical proof is a refinement of the natural logical capacity of our minds.
- Just as developing a discriminating ear through varied listening is essential preparation for understanding and playing music, developing logical discrimination through exposure to varied arguments is essential preparation for understanding and doing mathematics.
- The following exercise will do exactly that!

Instructions

- You will be presented with a succession of arguments and asked to quickly decide which are valid and which are invalid. Number successive lines on a piece of paper as instructed. When presented with each argument, write "V" next to its number if it is valid, "I" if it is invalid.
- You will have only a few seconds. Do not attempt to analyze the arguments formally. Instead, make an immediate judgement using your natural logical intuition.
- After each argument, you will be shown whether it is valid or invalid, but do not correct your answer if it was wrong. We will proceed immediately to the next example.
- We will repeat the exercise several times. If you made errors, your answers should improve with each attempt.
- At the very end, we will discuss each argument.

Instructions, Continued

- Interpret each argument as universal with respect to all unknown quantities. For example, the statement, "If it is a smurf, then it is blue," applies to all possible things that could replace the word "it."
- Each argument is self contained. Do not assume any information from the other arguments or from general knowledge. Base your answer only on what the argument literally says.

Argument 1

- If it is a smurf, then it is blue.
- Fred is a smurf.
- Therefore, Fred is blue.

Answer

Valid

Argument 2

- If it is a smurf, then it is blue.
- Fred is blue.
- Therefore, Fred is a smurf.

Answer

Invalid

Argument 3

- All smurfs are blue
- Fred is a smurf.
- Therefore, Fred is blue.

Answer

Valid

Argument 4

- All smurfs are blue.
- Fred is blue.
- Therefore, Fred is a smurf.

Answer

Invalid

Argument 5

- Some smurfs are blue.
- Fred is a smurf.
- Therefore, Fred is blue.

Answer

Invalid

Argument 6

- Some smurfs are blue.
- Fred is not blue.
- Therefore, Fred is not a smurf.

Answer

Invalid

Argument 7

- All smurfs are blue.
- Fred is not blue.
- Therefore, Fred is not a smurf.

Answer

Valid

Argument 8

- All smurfs are either blue or green.
- Fred is a smurf.
- Fred is not blue.
- Therefore, Fred is green.

Answer

Valid

Argument 9

- Some smurfs are blue.
- Fred is a smurf.
- Fred is not blue.
- Therefore, there are at least two distinct smurfs.

Answer

Valid

Argument 10

- All smurfs are blue, and if it is blue, then it is a smurf.
- Fred is blue.
- Therefore, Fred is a smurf.

Answer

Valid

Argument 11

- All smurfs are blue, and if it is blue, then it is a smurf.
- Fred is a smurf.
- Therefore, Fred is blue.

Answer

Valid

Argument 12

- All smurfs are either blue or green.
- If a smurf is blue, it likes chocolate.
- If a smurf is green, it likes apples.
- Fred is a smurf.
- Therefore, Fred likes either chocolate or apples.

Answer

Valid

Argument 13

- All smurfs are either blue or green.
- If a smurf is blue, it likes chocolate.
- If a smurf is green, it likes apples.
- Fred is a smurf.
- Fred does not like chocolate.
- Therefore, Fred is likes apples.

Answer

Valid

Argument 14

- If a smurf is blue, it likes chocolate.
- If a smurf is green, it likes apples.
- Fred is a smurf.
- Fred does not like chocolate.
- Therefore, Fred likes apples.

Answer

Invalid

Argument 15

- If a smurf is blue, it likes chocolate.
- If a smurf is green, it likes apples.
- Fred is a smurf.
- Fred likes apples.
- Therefore, Fred is green.

Answer

Invalid

Argument 16

- If a smurf is blue, it likes chocolate.
- If a smurf is green, it likes apples.
- Fred is blue.
- Therefore, Fred likes chocolate.

Answer

Invalid

Argument 17

- If a smurf is blue, it likes chocolate.
- If a smurf is green, it likes apples.
- Fred does not like chocolate, and Fred does not like apples.
- Therefore, Fred is not a smurf.

Answer

Invalid

Argument 18

- All smurfs are either blue or green.
- If a smurf is blue, it likes chocolate.
- If a smurf is green, it likes apples.
- Fred does not like chocolate, and Fred does not like apples.
- Therefore, Fred is not a smurf.

Answer

Valid

Argument 19

- All smurfs like apples or chocolate.
- Fred is a smurf.
- Fred likes apples.
- Therefore, Fred does not like chocolate.

Answer

Invalid

Argument 20

- All smurfs like apples or chocolate.
- Anything that likes apples is blue.
- Anything that likes chocolate is blue
- Therefore, all smurfs are blue.

Answer

Valid

Argument 20

- If it likes apples, then it does not like chocolate.
- Therefore, if it likes chocolate, then it does not like apples.

Answer

Valid

Argument 21

- All smurfs like apples or chocolate.
- If it is green, it likes apples.
- If it is blue, it likes chocolate.
- If it likes apples, it does not like chocolate.
- Therefore, if a smurf likes apples, then it is green.

Answer

Invalid

Argument 22

- All smurfs are either green or blue.
- If it is green, it likes apples.
- If it is blue, it likes chocolate.
- If it likes apples, it does not like chocolate.
- Therefore, if a smurf likes apples, then it is green.

Answer

Valid

Argument 23

- If a smurf is not green, it likes chocolate.
- If it likes apples, it does not like chocolate.
- Therefore, if a smurf likes apples, then it is green.

Answer

Valid

Argument 24

- If a smurf is green, it likes chocolate.
- If a smurf is not green, it likes chocolate.
- Therefore, all smurfs like chocolate.

Answer

Valid

Argument 25

- If a smurf is green, it likes chocolate.
- If a smurf is not green, it likes apples.
- Therefore, all smurfs like chocolate or apples.

Answer

Valid

Argument 26

- Anything red likes apples and chocolate.
- If a smurf likes apples, then it does not like chocolate.
- Therefore, no smurf is red.

Answer

Valid

Argument 27

- Anything red likes apples or chocolate.
- If a smurf likes apples, then it does not like chocolate.
- Therefore, no smurf is red.

Answer

Invalid