

Mon. 9/4 is Labor Day! 😊

Wed. 9/6 I. Open propositions define sets.

$$\text{Ex. } \{x \in \mathbb{R} : x = \sqrt{x+2}\}$$

$$\text{Ex. } \{x \in \mathbb{R} : x^2 - 3x < -2\}$$

$$\text{Ex. } \{x \in \mathbb{R} : x > 3\}$$

To get a simple expression for the first example, we solve the equation to get an equivalent proposition:

$$\begin{aligned}x &= \sqrt{x+2} \implies \\x^2 &= x+2 \iff \\x^2 - x - 2 &= 0 \iff \\(x+1)(x-2) &= 0 \iff \\x &= -1 \text{ or } x = 2\end{aligned}$$

Think about the reasons!

But only  $x=2$  is a solution.

Explain the logic!

$$\{x \in \mathbb{R} : x = \sqrt{x+2}\} = \{2\}.$$

To simplify the second description we solve the inequality:

$$\begin{aligned}x^2 - 3x < -2 &\Leftrightarrow x^2 - 3x + 2 < 0 \\ &\Leftrightarrow (x-1)(x-2) < 0 \Leftrightarrow \\ &[(x-1) < 0 \text{ and } (x-2) > 0] \text{ or } [x-1 > 0, x-2 < 0]\end{aligned}$$

$$\Leftrightarrow [x < 1 \text{ and } x > 2] \text{ or } [x > 1 \wedge x < 2].$$

$$\{x \in \mathbb{R} : x^2 - 3x < -2\} = \{x \in \mathbb{R} : 1 < x < 2\},$$

$$\text{Note: } \{x \in \mathbb{Z} : x^2 - 3x < -2\} = \emptyset.$$

II. Quantified statements:

$\neg \exists x \in \mathbb{R} : x < 1 \wedge x > 2$  is equivalent to

$\forall x \in \mathbb{R} : x \leq 1 \vee x \leq 2$ . (Can we use DeMorgan, but that goes beyond propos logic. How do we know this is true?)

Is it true that  $\forall x \in \mathbb{R} : 1 < x < 2$ ?

An important negation:

$$\forall x \in \mathbb{R}, x > 2 \Rightarrow x > 5.$$

Key:  $\exists x \in \mathbb{R} : x > 2 \wedge x \not> 5.$

3 is a counter-example.

In order to prove universal statements, we need universal axioms!

Fri  
9/8

Axioms of  $\mathbb{R}$ .

I. Algebraic Axioms: **Show slide**

II. Fundamental Properties:

a) Uniqueness of identity. ( $0$  +,  $1$   $\cdot$ )  
(Naming (specifying))

b) Uniqueness of inverses. (" $-$ ") (" $^{-1}$ ")

c) Multiplicative Property of  $0$ .

d) Non-existence of zero divisors.

$\mathbb{H}_4$  as an example of an algebraic system w/  $0$  divisors.

Importance  
for solving  
equations

# Assignments:

Asn. 3, Market exercises in sections 3.3, 3.5  
Draft due Mon 9/11, Final Due Fri 9/15.

Asn. 4 Market exercises in sec. 4.4,  
Draft due Mon, 9/18, Final due Fri 9/22.

Final Exam: Wed 9/27