MAT 2550, Assignment on Rank and Nullity

April 8, 2019

1. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}^2$ given by orthogonal projection onto the x, y-plane: $(x, y, z) \mapsto (x, y)$.
   (a) What is the dimension of Null(λ)?
   (b) What is the dimension of Ran(λ)?
   (c) Provide a basis for Null(λ).
   (d) Provide a basis for Ran(λ).

2. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}^3$ given by orthogonal projection onto the plane $\{(x, y, z) : x = y\}$.
   (a) What is the dimension of Null(λ)?
   (b) What is the dimension of Ran(λ)?
   (c) Provide a basis for Null(λ).
   (d) Provide a basis for Ran(λ).

3. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}$ given by orthogonal projection onto the x-axis: $(x, y, z) \mapsto x$.
   (a) What is the dimension of Null(λ)?
   (b) What is the dimension of Ran(λ)?
   (c) Provide a basis for Null(λ).
   (d) Provide a basis for Ran(λ).

4. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}^3$ given by orthogonal projection onto the line $\{(x, y, z) : x = y \& z = 0\}$.
   (a) What is the dimension of Null(λ)?
   (b) What is the dimension of Ran(λ)?
   (c) Provide a basis for Null(λ).
   (d) Provide a basis for Ran(λ).

5. Let λ be the linear map $\mathbb{P}_3 \to \mathbb{P}_3$ given by $\lambda(f) = f'$.
   (a) What is the dimension of $\text{Null}(\lambda)$?
   (b) What is the dimension of $\text{Ran}(\lambda)$?
   (c) Provide a basis for $\text{Null}(\lambda)$.
   (d) Provide a basis for $\text{Ran}(\lambda)$.
6. Let λ be the linear map $\mathbb{P}_3 \to \mathbb{P}_3$ given by $\lambda(f) = f''$.
   (a) What is the dimension of $\text{Null}(\lambda)$?
   (b) What is the dimension of $\text{Ran}(\lambda)$?
   (c) Provide a basis for $\text{Null}(\lambda)$.
   (d) Provide a basis for $\text{Ran}(\lambda)$.
7. Let λ be the linear map $\mathbb{P}_3 \to \mathbb{P}_3$ given by $\lambda(f) = f' + f''$.
   (a) What is the dimension of $\text{Null}(\lambda)$?
   (b) What is the dimension of $\text{Ran}(\lambda)$?
   (c) Provide a basis for $\text{Null}(\lambda)$.
   (d) Provide a basis for $\text{Ran}(\lambda)$.
8. Suppose M is the matrix for a surjective linear map. Prove that the rows of M_λ are linearly independent.
9. Suppose M_λ is the matrix for an injective linear map $\lambda : V \to W$. Prove that the rows of M_λ span V.
10. A few problems from the text: Section 4.2, # 36, 38, 40.