MAT 2550, Inverse Maps and Their Matrix Representations

April 15, 2020

To be handed in. Due by 5 p.m. on Friday, April 17, 2020.

Note: A possible answer to part (b) is "none of the above," in which case neither a right nor a left inverse exists and there is nothing "appropriate" to do for part (c).

1. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}^2$ given by orthogonal projection onto the x, y-plane: $(x, y, z) \mapsto (x, y)$.
   (a) Provide the matrix for λ in terms of the standard bases for $\mathbb{R}^3$ and $\mathbb{R}^2$.
   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?
   (c) Provide, as appropriate, a matrix, in terms of the standard bases for $\mathbb{R}^2$ and $\mathbb{R}^3$, for a left inverse, a right inverse, or the unique inverse of λ.
2. Let λ be the linear map $\mathbb{R}^2 \to \mathbb{R}^2$ given by rotation about the origin by $30°$.
   (a) Provide the matrix for λ in terms of the standard basis for $\mathbb{R}^2$.
   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?
   (c) Provide, as appropriate, a matrix, in terms of the standard basis for $\mathbb{R}^2$, for a left inverse, a right inverse, or the unique inverse of λ.
3. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}^3$ given by rotation about the z-axis by $45°$.
   (a) Provide the matrix for λ in terms of the standard basis for $\mathbb{R}^3$. [Hint: The z-coordinates do not change under this transformation; think of it as if you are looking from above at the xy-plane and rotating it about the origin.]
   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?
   (c) Provide, as appropriate, a matrix, in terms of the standard basis for $\mathbb{R}^3$, for a left inverse, a right inverse, or the unique inverse of λ.
4. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}^3$ given by orthogonal projection onto the plane $\{(x, y, z) : x = y\}$.
   (a) Provide the matrix for λ in terms of the standard basis for $\mathbb{R}^3$. [Hint: Break the transformation down into the composition of three transformations: first rotate the plane $x = y$ about the z-axis into the xy- or xz-plane (your choice), then project orthogonally onto that plane (which just requires changing the remaining coordinate to 0), then rotate back.]
   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?
   (c) Provide, as appropriate, a matrix, in terms of the standard basis for $\mathbb{R}^3$, for a left inverse, a right inverse, or the unique inverse of λ.
5. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}$ given by orthogonal projection onto the x-axis: $(x, y, z) \mapsto x$.
   (a) Provide the matrix for λ in terms of the standard bases for $\mathbb{R}^3$ and $\mathbb{R}$. (The standard basis for $\mathbb{R}$ is just the number 1.)

(b) Is λ injective (but not surjective), surjective (but not injective), or bijective?

(c) Provide, as appropriate, a matrix, in terms of the standard bases for $\mathbb{R}^2$ and $\mathbb{R}$, for a left inverse, a right inverse, or the unique inverse of λ.

6. Let λ be the linear map $\mathbb{R}^3 \to \mathbb{R}^3$ given by orthogonal projection onto the line $\{(x, y, z) : x = y \;\&\; z = 0\}$.

   (a) Provide the matrix for λ in terms of the standard basis for $\mathbb{R}^3$. [Hint: Two steps. First project onto $x = y$ using your answer to Exercise 2, then project orthogonally onto $z = 0$ (which is just the xy-plane). Drawing a picture will help you see that this composition does the right thing.]

   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?

   (c) Provide, as appropriate, a matrix, in terms of the standard basis for $\mathbb{R}^3$, for a left inverse, a right inverse, or the unique inverse of λ.

7. Let λ be the linear map $P_3 \to P_2$ given by $\lambda(f) = f'$. (Here f denotes a polynomial function.)

   (a) Provide the matrix for λ in terms of the standard bases for P_3 and P_2.

   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?

   (c) Provide, as appropriate, a matrix, in terms of the standard basis for P_3, for a left inverse, a right inverse, or the unique inverse of λ.

8. Let λ be the linear map $P_3 \to P_1$ given by $\lambda(f) = f''$.

   (a) Provide the matrix for λ in terms of the standard bases for P_3 and P_1.

   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?

   (c) Provide, as appropriate, a matrix, in terms of the standard basis for P_3, for a left inverse, a right inverse, or the unique inverse of λ.

9. Let λ be the linear map $P_3 \to P_3$ given by $\lambda(f) = f + f''$.

   (a) Provide the matrix for λ in terms of the standard basis for P_3.

   (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?

   (c) Provide, as appropriate, a matrix, in terms of the standard basis for P_3, for a left inverse, a right inverse, or the unique inverse of λ.

10. Let λ be the linear map $P_3 \to P_2$ given by $\lambda(f) = f' + f''$.

    (a) Provide the matrix for λ in terms of the standard bases for P_3 and P_2.

    (b) Is λ injective (but not surjective), surjective (but not injective), or bijective?

    (c) Provide, as appropriate, a matrix, in terms of the standard basis for P_3, for a left inverse, a right inverse, or the unique inverse of λ.