MAT 2443: Exam 2′ **Name:** ________________________________
March 28, 2014.

No calculators, notes, or books are allowed except for one index card. You may have only writing implements (including a ruler or other drawing aids) and blank paper.

Each numbered question is worth 20 points; any lettered parts of a question have the same value.

1. What point on the curve $\mathbf{r}(t) = (e^{-\frac{t}{2}}, e^{t}, e^{-\frac{t}{2}})$ is closest to the origin $(0, 0, 0)$?

2. Let $F : \mathbb{R} \to \mathbb{R}^3$ and $G : \mathbb{R} \to \mathbb{R}^3$ be differentiable functions, and suppose that $F(0) = (1, 0, 0)$, $F'(0) = (1, 1, 1)$, $G(0) = (0, 1, 0)$, and $G'(0) = (1, 1, 0)$. Evaluate $(F \times G)'(0)$ (where the function $F \times G$ is defined in the obvious way by $F \times G(t) = F(t) \times G(t)$).

3. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2 + y^2}$ does not exist.

4. What is the curvature at the point $(2, 0, 0)$ of the curve in which the elliptic cylinder $\frac{x^2}{4} + \frac{y^2}{9} = 1$ intersects the surface $z = y^3$?

5. (a) Write the equation of the tangent plane to the surface $z = e^{xy}$ at the point $(1, 1, e)$. You may use either local differential coordinates (dx, dy, dz) at $(0, 0, 1)$ or the ambient spacial coordinates (x, y, z), whichever you prefer. If you use local differential coordinates, indicate with a sketch or briefly describe in words what these coordinates represent.

(b) If $x^2 + y^2 + z^2 = 3$, calculate $\frac{dz}{dx}$ and $\frac{dz}{dy}$ at the point $(1, 1, 1)$.