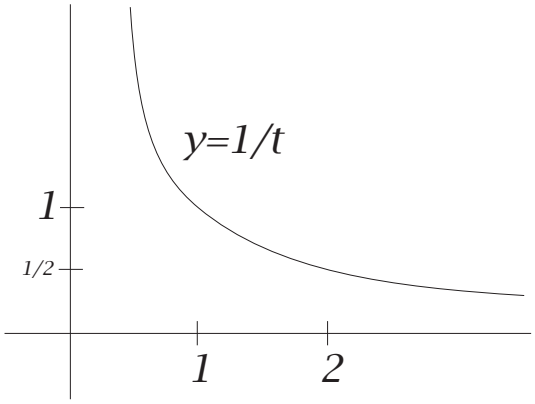
MAT 2442: Midterm Exam **Name:** _____
October 18, 2017

No calculators, notes, or books are allowed. You may have only writing implements and blank paper. Succinctly show the reasoning justifying each solution. Each numbered question is worth 20 points; all lettered parts have equal value.

1. (a) Use the definition of the natural logarithm as $\ln x = \int_1^x \frac{dt}{t}$ to show that $\ln 2 > \frac{1}{2}$. (Hint: Compare the area under the curve $y = \frac{1}{t}$ between $t = 1$ and $t = 2$ to the area of an appropriate rectangle.) Illustrate your answer with a picture on the graph below. A brief explanation is sufficient as long as the picture is correct!



(b) Evaluate: $\lim_{x \to \infty} x^{\frac{2}{x}}$

2. Evaluate:

(a) $\int_1^2 x \ln x dx$

(b) $\int_0^1 xe^x dx$

3. (a) $\int_0^1 \frac{dx}{\sqrt{9x^2+16}}$

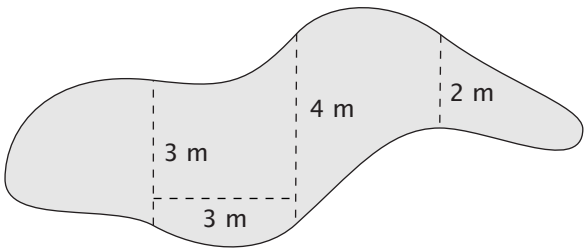
(b) $\displaystyle\int_0^1 \frac{x^3+x^2+2x+1}{(x^2+1)^2}dx$

4. (a) Evaluate or, if the integral diverges to infinity, write ∞: $\int_1^\infty e^{-x}dx$

(b) Evaluate or, if the integral diverges to infinity, write ∞: $\int_1^\infty \frac{dx}{x}$

(c) Determine if the following integral converges or diverges, and briefly justify your answer; do not attempt to evaluate it. $\int_1^\infty \frac{e^{-x}dx}{x}$

(d) Use Simpson's rule to approximate the area of the region depicted below. (Recall that the formula for Simpson's Rule is $\int_a^b f(x)dx \approx \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$, where n is the number of subintervals and $\Delta x = \frac{b-a}{n}$ is the length of each subinterval.)



5. (a) Compute the exact length of the curve $y = 1 + \frac{2}{3}x^{\frac{3}{2}}$, $0 \leq x \leq 1$.

(b) The curve $y = \frac{x^3}{3}$, $0 \leq x \leq 1$, is revolved about the x-axis. Calculate the area of the surface generated.