D. $\int \frac{x^3+2x^2+3x-2}{(x^2+2x+2)^2}\,dx$

Fri 9/22 Catch up, Questions, Quiz.

Mon 9/25

O. Go over quiz!

I. Questions on trig. substitution and partial fractions.

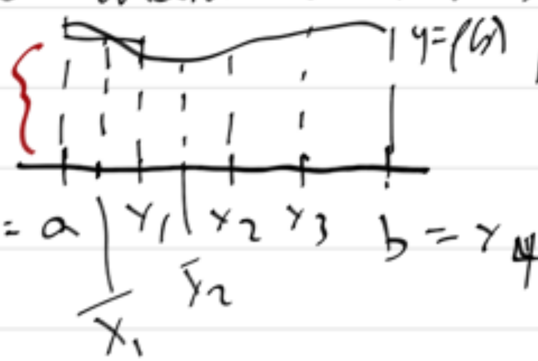
II. Try to get to some of

III. Sample Quiz

Tue. 9/26 Approximate Integration:

I. Three main methods:

A. $\bar{y}_i = f(\bar{x}_i)$



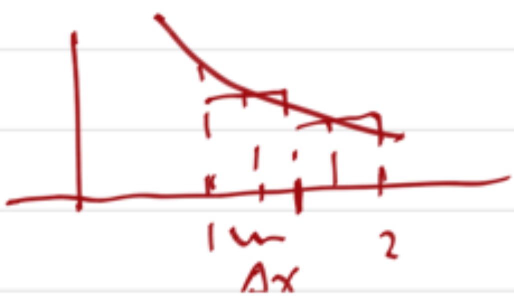
Let $\bar{x}_i = \frac{1}{2}(x_{i-1}+x_i)$

$\Delta x = \frac{b-a}{n}$

So $\int_a^b f(x)\,dx \approx$

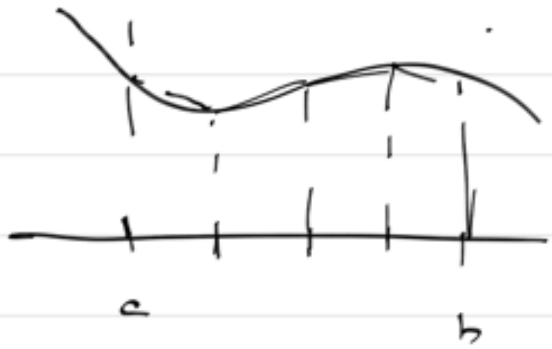
Let's do an actual example!

$\ln x = \int_1^x dx$, so let's estimate $\ln 2$ w/ $n=2$

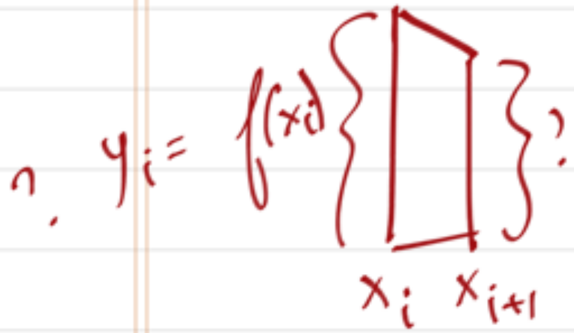


Underestimate or overestimate?

B. Trapezoidal:



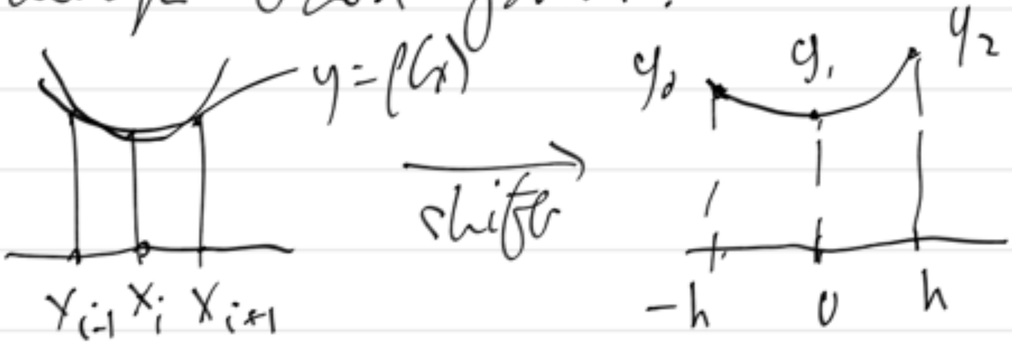
Area of trapezoid:



Now let's use $n=2$ to estimate $\ln 2 = \int_1^2 \frac{dx}{x}$ this way.

C. Simpson's Rule (Parabolic Approx.)

Given 3 points, we can always find a vertical parabola $y = Ax^2 + Bx + C$ through those points.



$h = \Delta x$

We only care about the area under this parabola.

This area is $\int_{-h}^{h}[Ax^2+Bx+C]\,dx =$

$$\frac{Ax^3}{3}+\frac{Bx^2}{2}+Cx\Big|_{-h}^{h} = \frac{2Ah^3}{3}+2C =$$

$\frac{h}{3}(2Ah^2+6C)$. Now note:

$$y_0 = Ah^2 - Bh + C$$
$$y_1 = C$$
$$y_2 = Ah^2 + Bh + C$$

So $y_0 + 4y_1 + y_2 = 2Ah^2 + 6C$.

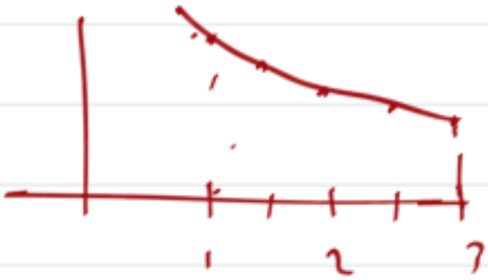
Thus, the area is $\frac{\Delta x}{3}(y_{i-1}+4y_i+y_{i+1})$,

and $\int_a^b f(x)\,dx = \frac{\Delta x}{3}(y_0+4y_1+2y_2+4y_3+\cdots$

$+4y_{n-1}+y_n)$.

Ok, let's do #2 this way w/ $n=2$.
(Only one parabola.)

For practice, let's also estimate $\ln 3$ w/ $n = 4$:



Wed 9/27

How good are these estimates?

I. M_n and T_n depend on the 2^{nd} derivative, which makes sense - Why?

Worst case scenario: Let $K^{(2)} =$ "Max" $\{|f^{(2)}(x)| : a \le x \le b\}$.

Ex. To compute the max. error in our estimate of $\ln 2$, $K^{(2)} = ?$

The formulas are
$$|E_M| \le \frac{K^{(2)}(b-a)^3}{24n^2}$$
$$|E_T| \le \frac{K^{(2)}(b-a)^3}{12n^2}$$

Suppose f is concave up. Which will be an under or over estimate? If f is concave down?

II. $|E_S|$ depends on the 4th derivative

(Parabola is more flexible than a circle).

Let $K^{(4)} =$ "Max" $\{|f^{(4)}| : a \le x \le b\}$.

What is this for our ln estimate?

$$|E_S| \le \frac{K^{(4)}(b-a)^5}{180n^4},$$

III. How large does n need to be to ensure $|E| \le .001$ in calculation of ln 2 for each rule?

Mix of my help and their work.

Note Desired bound $\le |E_s| \le K\cdots$.

Thurs. 9/28 I. Integrals over infinite intervals:

A. Ex. $\int_1^\infty \frac{dx}{x^2} \overset{\text{Def.}}{=} \lim_{b\to\infty} \int_1^b \frac{dx}{x^2} = \lim_{b\to\infty} \frac{-1}{x}\Big|_1^b$

$= \lim_{b\to\infty} \frac{-1}{b} + 1 = 1.$ Converges

Ex. $\int_1^\infty \frac{dx}{x} = \lim_{b\to\infty} \ln x\Big|_1^b = \lim_{b\to\infty} \ln b = \infty.$

Diverges.

Ex. $\int_1^\infty \frac{dx}{\sqrt{x}}$ Ex. $\int_1^\infty \frac{dx}{x^{3/2}}$

In general, if $p \neq 1$, what is

$\int \frac{dx}{x^p}$? When does $\int_1^\infty \frac{dx}{x^p}$ converge?

Calculate $\int_4^\infty \frac{dx}{x^2}$.

B. Sometimes we can tell an integral is convergent (i.e. finite) w/out being able to calculate its exact value:

Ex. $\int_0^\infty e^{-x^2}dx$. For $x \geq 1$, $e^{-x^2} \leq e^{-x}$,

and $\int_0^1 e^{-x^2}dx$ is finite (although we don't know exactly its value), so

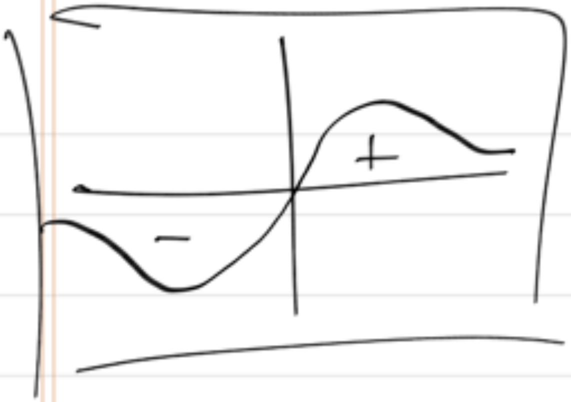
$$\int_0^\infty e^{-x^2}dx \leq \int_0^1 e^{-x^2}dx + \int_1^\infty e^{-x}dx$$

$$= \int_0^1 e^{-x^2}dx - e^{-x}\Big|_1^\infty = \int_0^1 e^{-x^2}dx + \frac{1}{e}.$$

C. If the interval is infinite at both ends, we split it into two parts and <u>both</u> must converge.
("Infinite cancellation is indeterminate")

Ex. $\int_{-\infty}^{\infty} \frac{dx}{1+x^2} < \infty$. Why won't changing the break point change the answer?

Ex. But $\int_{-\infty}^{\infty} \frac{x\,dx}{1+x^2}$ diverges, even though



II. Integrals over discontinuities.

Ex. $\int_0^1 \frac{dx}{\sqrt{x}} = \lim_{a\to 0^+} 2\sqrt{x}\Big|_a^1 = \lim_{c\to 0^+} 2-2\sqrt{c} = 2.$

Remark: $y = x^{-1/2} \Leftrightarrow x = y^{-2}$. Symmetric!

$$\int_0^1 \frac{dx}{x} = \lim_{a\to 0^+} \ln x\Big|_a^1 = +\infty \text{ Diverges.}$$

In general, if $p\neq 1$, $\int_0^1 \frac{dx}{x^p} = \lim_{a\to 0^+} ?$, so

$$\int_0^1 \frac{dx}{x^p} \begin{cases} & , \text{ if } p<1 \\ & , \text{ if } p\geq 1 \end{cases}$$

Fri 9/29 I. Catch up

II. Questions for Exam:

Determine convergence or divergence by comparison to known integrals:

A. $\int_1^\infty \frac{e^{-x}dx}{x}$ B. $\int_0^1 \frac{e^{-x}dx}{\sqrt{x}}$ C. $\int_0^1 \frac{e^{-x}dx}{x}$

III. Quiz Allow 25-30 min!