Mon is Labor Day! ☺

Post Assn. 3!

Tues. 9/5
I. Go through Slides 28-30.
II. Do marked problems on Assn. 2.
Sample Quiz 2 tomorrow! Finish Assn. 2!

Wed. 9/6
I. Answer questions on Assn. 2
II. Integration by parts:

$$d(uv) = u\,dv + v\,du \iff$$
$$uv = \int u\,dv + \int v\,du + C \iff$$
$$\int u\,dv = uv - \int v\,du + C.$$

Note: since we only know dv, v is only determined up to a constant, but

$$u(v+D) - \int (v+D)\,du + C$$
$$uv + Du - \int v\,du - D\int du + C$$
$$uv + Du - \int v\,du - Du + C,$$

Ex. (a) $\int_0^1 x e^x\,dx =$

$$x e^x\Big|_0^1 - \int_0^1 e^x\,dx = \dots$$

Let $u = x$, $dv = e^x dx$

$du = dx$, $v = e^x$

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$= \frac{du}{v} - \frac{u}{v^2}\,dv \implies$$

$$\frac{1}{v}\,du = \frac{u\,dv}{v^2} + d\left(\frac{u}{v}\right) \implies$$

$$\int \frac{1}{v}\,du = \int \frac{u\,dv}{v^2} + \frac{u}{v}$$

Not generally useful!

(b) Now, class, do: $\int_0^1 x^2 e^x dx$

(c) $\int_1^2 \ln x \, dx$

(d) Now, class, do $\int_1^2 x \ln x \, dx$

(e) $\int_1^2 x e^{x^2} dx$

III. Sample Quiz

Th. I. Questions on H.W.

9/7 II. **Some more difficult examples:**

(a) You should know: If $x<0$, then $\ln|x| = \ln(-x)$ makes sense, and $\frac{d(\ln|x|)}{dx} = \frac{1}{(-x)} \cdot (-1) = \frac{1}{x}$. So $\frac{d(\ln|x|)}{dx} = \frac{1}{x}$

Explain graphically how!

whether $x<0$ or $x>0$. (Careful! $x=0$ is forbidden!) So $\int \frac{dx}{x} = \ln|x| + C$ for $x \neq 0$.

(b) $\int \tan x \, dx = \int \frac{\sin x \, dx}{\cos x} = -\ln|\cos x|$,

$x \neq \pi/2, 3\pi/2, \ldots$

(c) Now, clam, ck $\int_c^{\pi/4} x\tan^2 x\,dx$. [Hint: $\tan^2+1=\sec^2$. (Remb why?)] So

$\int_0^{\pi/4} x\sec^2 x\,dx - \int_0^{\pi/4} x\,dx$ ← Easy!

$= x\tan x\Big|_0^{\pi/4} - \int_0^{\pi/4}\tan x\,dx$

$= \ldots$

let $u = x$ ⇓ $du = dx$ | $dv = \sec^2 x\,dx$ ⇑ $v = \tan x$

(d) A tricky integral you must learn:

$$\int \sec x\,dx = \int \frac{\sec x(\sec x+\tan x)}{(\sec x+\tan x)}\,dx = \ln|\sec x+\tan x| + C$$

(e) Now here's a hard one that introduces a new strategy:

$$\int \sec^3 x\,dx = \int \sec^2 x \sec x\,dx$$

$\sec^2 = \tan^2+1$ | $\tan' = \sec^2$

Let them close!

$$\int \sec^2 x \sec x \, dx =$$

$$\sec^2 x \tan x - \int \sec^2 x \tan^2 x \, dx$$

Hmm!

Let $u = \sec x$, $dv = \sec^2 x \, dx$

$du = \sec x \tan x$, $v = \sec x \tan x$

$$\int \sec^3 x \sec x \, dx =$$

$$\int (\tan^2 x + 1) \sec x \, dx =$$

$$\int \tan^2 x \sec x \, dx + \int \sec x \, dx =$$

$$\int \tan x \sec x \tan x \, dx + \ln|\sec x + \tan x|$$

Let $u = \tan x$, $du = \sec^2 x \, dx$

$dv = \sec x \tan x \, dx$, $v = \sec x$

$$= \sec x \tan x - \int \sec^3 x \, dx + \cdots$$

$$\Rightarrow 2\int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$$

$\Rightarrow \cdots$ Let you finish!

Finally, an easy one for Thu:

$\int \sec^2 x\,dx$.

And what about $\int \sec^4 x\,dx$?

$$\int \sec^4 x\,dx = \int \sec^2 x\,(\tan^2 x + 1)\,dx = \int \sec^2 x \tan^2 x\,dx + \int \sec^2 x\,dx$$

$$= \tan^3 x - 2\int \tan^2 x \sec^2 x\,dx$$
$$= \tan^3 x - 2\int \sec^4 x + 2\int \sec^2 x\,dx$$

Let $u = \tan^2 x$ ⇓ $du = 2\tan x \sec^2 x$

$dv = \sec^2 x\,dx$ ⇑ $v = \tan x$

Let them struggle w/ this!

Fri. 9/8 Catch up, Questions, and Quiz!