

Written Assignment 2

Due by the beginning of class on Tuesday, September 5, 2017.

1 The Natural Exponential Function and the Number e

1. Your employer says you will get the same percentage raise every year for the next five years, and you can negotiate what that percentage will be. Your annual salary is currently 50,000. You would like your annual salary to be 60,000 after the five raises. What percentage increase each year do you need to get? Round your answer to the nearest tenth of a percent. *Fully explain and justify your solution.*
2. Use implicit differentiation, the chain rule, and the fact that $\ln(e^x) = x$ to derive the derivative of the exponential function.
3. What is $\lim_{x \rightarrow +\infty} e^x$? What is $\lim_{x \rightarrow -\infty} e^x$? Give as many explanations for your answers as you can. Sketch the graph of $y = e^x$.
4. Explain using areas and the definition of the natural logarithm why $\ln 2 < 1$. (Hint: See Assignment 2, Section 1, Exercise 8.) Explain further why it follows that $e > 2$.
5. Explain using areas and the definition of the natural logarithm why $\ln 3 > 1$. (Hint: Use eight rectangles. You can also use midpoint sums with a smaller number of rectangles, but that requires a more refined argument.) Explain further why it follows that $e < 3$.
6. Let L and k be positive constants. Let t_0 be any real constant. The function defined by

$$f(t) = \frac{L}{1 + e^{-k(t-t_0)}}$$

is called a *logistic* function. To make the following calculations simple, let us consider the case that $L = k = 1$ and $t_0 = 0$.

- (a) Let $f(t) = \frac{1}{1+e^{-t}}$. Compute the formula for $f'(t)$.
- (b) Where is f increasing? Decreasing?
- (c) Compute the second derivative of f .
- (d) Does f have any inflection points? If so, where?
- (e) Sketch the graph of f .

2 Using the Natural Logarithm and Exponential Functions to Compute Limits and Derivatives

1. Evaluate $\lim_{x \rightarrow 0^+} x^x$.
2. Evaluate $\lim_{x \rightarrow 0^+} x^{\sin x}$.
3. Evaluate $\lim_{x \rightarrow +\infty} x^{\frac{1}{x}}$.
4. Evaluate $\lim_{x \rightarrow +\infty} \left(\frac{x^2}{x^2 + x} \right)^x$.
5. If $y = (x^2 + 1)^{(x^2+1)}$, find a formula for y' .
6. If $y = \frac{(x + 1)^8(x^2 + 1)^9}{(x - 1)^8(x^2 - 1)^9}$, find a formula for y' .

3 Additional Problems for Practice (as needed).

1.4 # 20, 24.

1.5 # 8, 10, 14, 16, 18, 20, 26, 36, 40, 50, 52, 64, 66, 68.