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1)(25pts) Solve the differential equations with initial conditions $y(0) = 1, y'(0) = 1, y''(0) = 1$ using Laplace transforms.

$$y''' = x - H_1(x)x$$

2)(25pts) Solve the differential equations with initial conditions $y(0) = 1, y'(0) = 0, y''(0) = 0, y'''(0) = 1$ using Laplace transforms.

$$y'''' = x - \sum_{n=1}^{\infty} H_n(x)$$

3) (25pts) Use the convolution method to compute:

$$\mathcal{L}^{-1}\left[\frac{e^{-3s}}{(s-1)(s-2)}\right]$$

4) (25pts) Compute the Laplace transform for:

$$\mathcal{L}\left[\sum_{n=1}^{\infty} H_n(x)x^2\right]$$