Challenges of the Week
Spring Semester 2008-2009

Challenge of the Week # 1 - January 21 to January 30: A street hustler is trying to take your money. He has four cards in front on him. The first is a Jack, the second a Queen, the third a King and the last card is an Ace. He turns over all cards. He interchanges the first and third card, then the second and third and finally the first and last cards. He repeats this process of three interchanges 19 times. What is the final location of the Queen? Justify your answer.

Challenge of the Week # 2 - January 30 to February 6: Consider the following process:

> *A positive integer n is chosen. Then the product $n(n+1)$ is computed and two digits are appended to the end of the product. The resulting number is the square of an integer.*

Show that it is always possible to complete the process above or give a positive integer n for which the process cannot be completed.

Challenge of the Week # 3 - February 6 to February 13:

This is a very challenging challenge

Two distinct positive integers are chosen. The square of the smaller integer is computed. The larger of the two original integers is replaced with the absolute difference of the two original integers. The process continues with the absolute value and the original smaller integer. From these, the square of the smaller integer is computed and the larger integer is again replaced with the absolute value of the difference. This process is continued until the difference is zero. What is the sum of the squares that have been computed? Justify your answer.

Challenge of the Week # 4 - February 20 to February 27: Several real numbers (not necessarily all different) are chosen. The sum of these numbers is 10. Is it possible that the sum of the squares of these numbers is less than one-millionth? Justify your answer.

Challenge of the Week # 5 - February 27 to March 6:

1. *Prove there exists a convex 10-gon that has 3 acute interior angles.*
2. *Prove there does not exist a convex 10-gon that has 4 acute interior angles.*

Challenge of the Week # 6 - March 6 to March 13: Find the smallest positive integral value of n for which the following product is an integer.

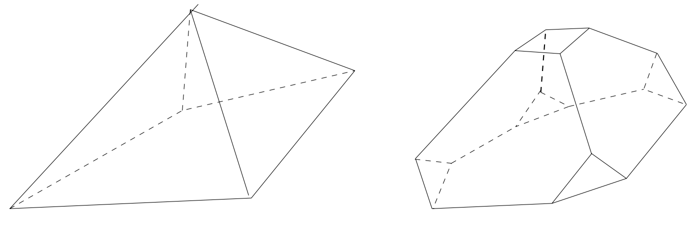
$$\left(1\frac{1}{36}\right)\left(1\frac{1}{37}\right)\cdots\left(1\frac{1}{n}\right)$$

Justify your answer.

Challenge of the Week # 7 - March 23 to April 3: This is the annual April Fool's Challenge of the Week.

1. *A clock strikes six times in five seconds. How many times will it strike in 10 seconds? Justify your answer.*
2. *Let γ be a closed non-intersecting curve all of whose points are 1 inch from a given point. Is it possible that γ is longer than 1 mile? Justify your answer.*
3. *Let $\triangle ABC$ be a triangle all of whose sides are shorter than 1 inch. Can the circle which passes through A, B, and C have circumference which is larger than 1 mile? Justify your answer.*

Challenge of the Week # 8 - April 3 to April 10: A small pyramid is chopped off of each vertex of a convex polyhedron $\mathcal{P}$. The result is a truncated polyhedron $\mathcal{T}$. This process is illustrated below when $\mathcal{P}$ is a square pyramid.



Suppose the original polyhedron $\mathcal{P}$ had 100 edges.

- *How many edges are there in $\mathcal{T}$?*
- *How many vertices are there in $\mathcal{T}$?*

Justify your answers.

Challenge of the Week # 9 - April 10 to April 17: Let A be the number which consists of 2009 digits all equal to 2. Let B be the number which consists of 2009 digits all equal to 3. Compute $A + B^2$ and explain your computation.

Challenge of the Week # 10 - April 17 to April 24: Two nonzero real numbers are chosen. Which is larger — the cube of the sum of the squares of the the two numbers or the square of the sum of the cubes of the two numbers? Or, is it that in some cases the cube of the sum of the squares is larger and in other cases the square of the sum of the cubes is larger? Justify your answer.