Challenges of the Week
Solutions
Spring Semester 2002-2003

Challenge of the Week # 1 - January 24 to January 31: Let N be the number of people on Earth that made an odd number of handshakes by the end of 2002. Is it possible to deduce whether N is an even number or an odd number? Select which one of the following is true and explain why.

a) It is not possible to deduce whether N is even or odd.
b) N must be an odd number.
c) N must be an even number.

Complete solutions submitted by Bradley Ashby, Brad Heller, Jessie Murray, and Shane Reichart ($10 prize). Partial solutions submitted by Kelly Best, Maggie Granrath, Leah Hefner, Shannon Price, Symon Shmukler, Megan Sparks, and Johnathon Walker. Other submissions from Jenny Bauer, Karissa Berg, Jennifer Eakley, Panagiotis Efstathiou, Brent Furrow, Julie Kleinschmidt, Jason Luhr, Anna Pougas, Laura Rimington, Nick Upton, and Sarah Weiss. Solution: (Due to Shane Reichart) Each person involved in a handshake is regarded as having made a handshake. Therefore, the total number of handshakes at any given time must be an even number T.

N is the given number of people that made an odd number of handshakes. Define $\overline{N}$ to be the number of handshakes made by the N people that shook hands an odd number of times. $\overline{N}$ is a sum of N odd numbers, so if $\overline{N}$ is even then N is even and if $\overline{N}$ is odd then N is odd. Define M to be the number of handshakes made by those that shook hands an even number of times. Thus, M is a sum of even numbers so M is even.

We have that $T = M + \overline{N}$. Since T and M are even, $\overline{N}$ must be even which implies N is even.

An alternate, but slightly longer method of solving this problem is to analyze how the numbers in each group change when two people shake hands. Three solvers used this method.

Challenge of the Week # 2 - January 31 to February 7: Two students, Alice and Brandon, each think of a positive integer and they each tell their number to a professor without the other student hearing. The professor then secretly writes the sum of the two numbers on one piece of paper and writes the product of the two numbers on a second piece of paper. One of the papers is destroyed and the other paper, which has the number 2002 written on it, is shown to both students. The professor asks Alice if she can deduce Brandon's number. She says "No" and Brandon hears this. The professor then asks Brandon if he can deduce Alice's number. He also says "No". What is Brandon's number? Find all possible answers and justify your solution.

Complete solutions submitted by Brent Furrow, Shane Reichart ($10 prize), and Robert Schlacks. Partial solutions submitted by Maggie Granrath, Brad Heller, Jessie Murray, Diwakar Singh Thapa, and Travis White. Other submissions from Jenny Bauer, Kelly Best, Leah Hefner, Kimberly Kimbrough, Julie Kleinschmidt, Anna Pougas, Julia Smith, Sharon Tuttle, and Nick Uptown. Solution: (Due to Shane Reichart) Since Alice does not know Brandon's number, her number must be a factor of 2,002 (If it isn't, then she would know that 2,002 is the sum and therefore know Brandon's number). For the same reason, Brandon's number is also a factor of 2,002.

Because Brandon still doesn't know Alice's number, he must be uncertain whether 2,002 is the product or the sum. Therefore he must think that it is possible to achieve 2,002 by adding his number to another factor of 2,002. The factors of 2,002 are 1, 2, 7, 11, 13, 14, 22, 26, 77, 91, 143, 154, 182, 286, 1001, 2002. It is only possible to add two of these factors and get 2,002 if the two factors are 1,001. Thus Brandon's number must be 1,001.

We also know that if 2,002 is the product then Alice's number is 2 and if 2,002 is the sum, then Alice's number is 1,001. The paper that was destroyed had either 1,003, or $1001^2 = 1{,}002{,}001$ written on it.

Challenge of the Week # 3 - February 7 to February 13: It takes 17 minutes to cut a log into 19 equal parts. Suppose that it takes a whole number of minutes strictly between 40 minutes and 1 hour to cut the log into N equal parts. How long will it take to cut the log into 2N parts? Find the exact number of minutes that it takes to cut the log into 2N parts. (All cuts are identical and are perpendicular to the axis of the log.)

Complete solutions submitted by Kevin Coulton, Shannon Price, Shane Reichart, Robert Schlacks ($5 prize), Symon Shmukler, Julia Smith ($5 prize), and Diwakar Singh Thapa. Other submissions from Jenny Bauer, Kelly Best, Brent Furrow, Leah Hefner, Julie Kleinschmidt, Christine Piotrowski,

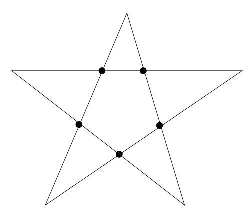
Anna Pougas, Brandi Reynolds, Laura Rimington, and Travis White. Solution: (Due to Shane Reichart) To cut log into N parts requires N-1 cuts. It is given that it takes 17 minutes to cut the log into 19 parts so this is a rate of 17min/18cuts or 17/18 minute per cut. Since cutting into N parts takes a whole number of minutes between 40 minutes and 1 hour we have that $40 < \frac{17(N-1)}{18} < 60$ where $\frac{17(N-1)}{18}$ is a whole number. This implies that $N - 1 = 54$ so $N = 55$. Then $2N = 110$ with 109 cuts. The time required is $\frac{17}{18}(109) = 102\frac{17}{18}$ minutes or 102 minutes and 56 2/3 seconds.

Challenge of the Week # 4 - February 21 to February 27: As shown in the figure, it is possible to draw a sequence of 5 line segments with each one starting where the last one ended and the last segment ending at the start of the first segment. Such a sequence is called a closed broken line. The example in the figure also has the property that each segment intersects exactly two of the other segments.

a) Is it possible to draw a closed broken line with 17 line segments such that each segment intersects exactly two of the other segments?

b) Is it possible to draw a closed broken line with **six** *segments such that each segment intersects exactly* **one** *of the other segments?*

c) Is it possible to draw a closed broken line with **seven** *segments such that each segment intersects exactly* **one** *of the other segments?*



Complete solutions submitted by Anuj Bhandari,and Shane Reichart ($10 prize). Partial solutions submitted by Brad Heller. Other submissions from Jenny Bauer, Julie Kleinschmidt, Megen Lindley, and Sarah Weiss. Solution: a) Yes it is possible. Draw 17 points on a circle and connect every other one. This same method can be used for any odd number of segments.

b) Yes it is possible. See figure 1. This can be generalized for any even number of line segments as shown in figure 2.

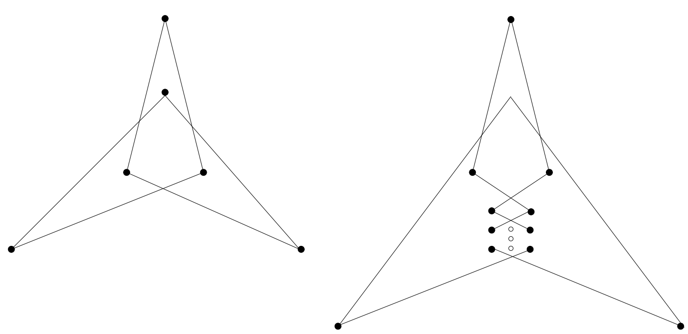


Figure 1 Figure 2

c) No it is not possible. Each intersection requires two intersecting line segments. Thus the number of segments equals twice the number of intersection points. This means that the number of segments must be an even number. 7 is not even so it is not possible.

Challenge of the Week # 5 - February 28 to March 7: On a small island there are 37 gray, 41 brown, and 55 purple chameleons. They always keep the same color except when two chameleons which have different colors meet, they both change their colors to the third color.

a) Could it happen that eventually all of the chameleons will have the same color?

b) If there are 37 gray, 41 brown, and 54 purple chameleons, could it happen that eventually all of the chameleons will have the same color?

If it could happen, then show how it could happen. If it cannot happen, explain why it cannot happen.

Complete solutions submitted by Kelly Best, Anuj Bhandari, and Shane Reichart ($10 prize). Partial solutions submitted by Kevin Coulton, Brad Heller, Symon Shmukler, Julia Smith, and Travis White. Other submissions from Jenny Bauer. Solution: Let g, b, and p be the numbers of gray, brown, and purple chameleons respectively. Consider the differences $g-b$, $g-p$, and $b-p$. If a gray and a brown chameleon meet, then the numbers will change to $g-1$, $b-1$, and $p+2$ with differences $g-b$, $g-p-3$, and $b-p-3$. Thus when two chameleons meet, the differences either do not change or they change by 3. To end up with all chameleons the same color, two of the colors will end up with zero chameleons so the difference would be zero. Since the difference can only be changed by 3, in order to get zero, the difference must start out being a multiple of three. In part b) the differences are -4, -17, and -13. None of these are divisible by 3 so it is not possible to end up with all chameleons the same color. In part a) $g-p=-18$ which is divisible by 3 so it might be possible to end up with the same color. It is possible as follows: First, 6 brown meet 6 purple which results in $g=49$, $b=35$, and $p=49$. Then 49 gray meet 49 purple which will result in all brown chameleons.

This completes the problem, but it would be nice to know if it is always possible to end up with all chameleons the same color whenever the difference between two colors is divisible by three. If two of the values are equal, then the corresponding chameleons could meet and they would then all be the same color, so without loss of generality, suppose $g>p$ and $g-p$ is divisible by 3. Then if there are $|g-p|/3$ brown chameleons, they can meet $|g-p|/3$ purple chameleons, and the number of gray chameleons will gain 2/3 of the difference $|g-p|$ so the number of gray will equal the number of purple. Then if all the gray meet all of the purple, the chameleons will all be brown.

If there are fewer than $|g-p|/3$ brown chameleons, we note that we may assume that $p=0$ since if not, p purple chameleons may meet p gray chameleons which will result in $p=0$. Thus, we need to show what to do in the case that $p=0$, g is divisible by 3, and $b<(g-p)/3=g/3$. In this case, have all of the brown chameleons meet b of the gray chameleons and then have all of the purple chameleons meet $2b$ of the gray chameleons. The triples change as follows: $(g,b,0)\to(g-b,0,2b)\to(g-3b,4b,0)$. Note that since $b<g/3$ we have that $g-3b>0$ and we are in the same situation as before, but g has decreased by $3b$ and b has increased by a factor of 4. Thus, now either $b\geq g/3$ or we can repeat this process until it is.

Thus, we have the general result: Given g, b, and p gray, brown, and purple chameleons respectively, it is possible for them all to eventually be the same color if and only if at least one of the three differences $g-b$, $g-p$, and $b-p$ is divisible by 3.

Challenge of the Week # 6 - March 17 to March 21: Let M be the number of Mondays, T the number of Tuesdays, and W the number of Wednesdays in a year. Could it happen in some year that

1) $M\neq T$, $T\neq W$, *and* $W\neq M$?

2) $M=T<W$?

3) $M=T>W$?

4) $M=W<T$?

5) $M=W>T$?

Explain your answers.

Complete solutions submitted by Shane Reichart ($10 prize). Partial solutions submitted by Jenny Bauer, Kelly Best, Abby Lyons, Julia Smith, Symon Shmukler, and Megan Sparks. Other submissions from Anuj Bhandari, Lydia Bruns, Brent Furrow, Christine Piotrowski, Sharon Tuttle, and Sarah Weiss. Solution: (due to Shane Reichart) Every week adds exactly one day to each day of the week, so we can ignore the first 364 days of the year since 364 is evenly divisible by 7. A year has one extra day (over 364) or two extra days on a leap year. Note that since years can start on any day of the week, the extra day or days can end on any day of the week. Also note that on a leap year the two extra days are adjacent.

1) No, it is not possible that $M\neq T$, $T\neq W$, and $W\neq M$ because there can be at most only a difference of 1 between two days of the week.

2) Yes, if the extra day of the year falls on a Wednesday, then $M=T<W$.

3) Yes, if the 2 extra days of a leap year fall on Monday and Tuesday, then $M = T > W$.
4) Yes, if the extra day of a year falls on Tuesday, then $M = W < T$.
5) No. Because the extra 2 days in a leap year must be adjacent it is impossible for one of these days to fall on a Monday and for the other to fall on a Wednesday. Thus it is not possible to have $M = W > T$.

Challenge of the Week # 7 - March 24 to March 28: There are 17 equally spaced balls on the left side of a horizontal line that are each moving at constant speed s to the right. There are also 19 equally spaced balls on the right side of the same line that are each moving at constant speed s to the left. The balls are all the same size and mass and the spacing between the balls on the left is the same as the spacing between the balls on the right. When two balls collide they will each reverse directions but each will maintain the same speed s. How many collisions will occur in the described system? Justify your answer.

Complete solutions submitted by Julie Kleinschmidt, Shane Reichart ($10 prize), Symon Shmukler, Julia Smith, and Travis White. Other submissions from Jenny Bauer, Brent Furrow, and Christine Piotrowski. Solution: The number of collisions in the system equals the product of the number of balls in the initial group moving to the left with the number of balls in the initial group moving to the right. In the above case, $17 \cdot 19 = 323$ total collisions.

To see why this is true, note that when two balls collide, since they each reverse direction, but are the same in all other respects, we can imagine that they have just passed through each other. Thus the number of collisions is the same as the number of balls moving left that we imagine passing through the balls moving right. Each left-moving ball passes through each right-moving ball so the total number of collisions is the product of the two numbers.

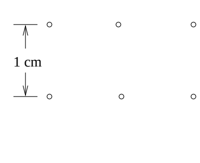
Another interesting way to solve this problem is to put 19 minus signs on the right and 17 plus signs on the left which each indicate a ball and the direction that it is going. When there is a collision the two balls reverse directions so an adjacent plus-minus is changed to a minus-plus. Several of the initial collisions are shown with this notation below.

+ + + + + + + + + + + + + + + + + - - - - - - - - - - - - - - - - - - -
+ + + + + + + + + + + + + + + + - + - - - - - - - - - - - - - - - - - -
+ + + + + + + + + + + + + + + - + - + - - - - - - - - - - - - - - - - -
+ + + + + + + + + + + + + + - + - + - + - - - - - - - - - - - - - - - -
+ + + + + + + + + + + + + - + - + - + - + - - - - - - - - - - - - - - -
etc.

Note that this can be thought of as having all of the plus signs move right and passing through all of the minus signs which are moving left. So again the total number of collisions is the product of the two numbers.

Challenge of the Week # 8 - March 28 to April 4:

1. *Go to* `http://www.charley-summerhill.com/psychic3.html` *and see if the Flash Mind Reader can read your mind. Explain how it works. (If you cannot view this page, you need Macromedia Flash Player. To get it, go to* `http://www.macromedia.com/downloads/`*.)*
2. *Find the value of the product* $\sin\alpha \cdot \sin\beta \cdot \ldots \cdot \sin\omega$*, where all of the letters of the Greek alphabet are used.*
3. *What is a YES-NO question for which the answer is always NO?*
4. *The centers of nine circles with radius* 10^{-6} *mm each form a 3 by 3 grid with 1 cm between two neighboring centers (see the figure). Draw the smallest number of line segments, without lifting your pencil, that intersect all nine circles.*



Complete solutions submitted by Anna Pougas ($10 prize), and Shane Reichart. Partial solutions submitted by Jamie Baker, Jenny Bauer, Elizabeth Baum, Anuj Bhandari, Brent Furrow, Jil Gates, Leah Hefner, Brad Heller, Julie Kleinschmidt, Andrea Niemerg, Christine Piotrowski, Jennifer Pitner, Symon Shmukler, Megan Sparks, Diwakar Singh Thapa, Sharon Tuttle, and Nick Upton. Solutions:

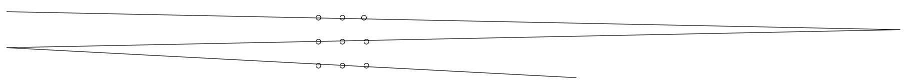
1) At the web site you will see a table of integers from 00 to 99 with a sign or figure to the right of each integer. The instructions are: "Pick any integer, subtract from it the sum of its digits, look at the sign next to the difference, and then click the magic glass.". When this is done, the computer shows the sign that you saw next to the difference. This may be surprising since you have not given the computer any information about which number you originally chose. The explanation becomes clear if you calculate a little.

Let $\overline{xy}$ be the number that you thought of on the table. Note that $\overline{xy} = 10x + y$. Subtracting the sum of the digits, $x+y$, yields $\overline{xy} - (x+y) = 10x + y - x - y = 9x$. Thus the answer is nine times the first digit x. For example, $53 - 8 = 9 \cdot 5$, $35 - 8 = 9 \cdot 3$, $13 - 4 = 9 \cdot 1$, $07 - 7 = 9 \cdot 0$, and so on. The secret to the table of signs is that the signs next to all of the integers that are divisible by 9, i.e. next to 0, 9, 18, 27, 36, 45, 54, 63, 72, 81 are the same (90 and 99 are not required since nine times the first digit is never larger than 81). Thus no matter which number you choose, the sign will be the sign that is next to one of the given multiples of 9, so that is the sign that is shown.

2) $\sin\alpha \cdot \sin\beta \cdot \ldots \cdot \sin\omega = 0$ since one of the factors is $\sin\pi$ which equals zero.

3) The following are some solutions. a) Are you dead? b) Are you asleep? c) Is 1 greater than 2? d) Can you move faster than light? e) Is there any other answer for a YES-NO question other than yes or no? f) Is this not a YES-NO question?

4) Three line segments:



Challenge of the Week # 9 - April 4 to April 11: Suppose that a satellite above the Earth can spy on all surface points that have an unobstructed view from the satellite (ignore clouds). Thus, the visible part of the surface of the Earth viewed from the satellite is strictly less than a hemisphere. What is the least number of such satellites, positioned around the Earth, required for spying on the entire Earth so that each point on the Earth is visible from some satellite? Find the smallest number of such satellites and justify that no smaller number suffices.

There were no complete solutions. Partial solution submitted by Shane Reichart ($10 prize). Other submissions from Jamie Baker, Jenny Bauer, Anne Beil, Kelly Best, Anuj Bhandari, Tony Ciolino, Brent Furrow, Julie Kleinschmidt, Julia Smith, Megan Sparks, Sharon Tuttle, Sarah Weiss, and Nick Upton. Solution: The least number of such satellites is 4.

First we prove that 4 satellites is sufficient for viewing the whole Earth. Let ABCD be a triangular pyramid that contains the Earth in its interior. For example, ABCD could be a regular circumscribed tetrahedron. If the satellites are at the 4 vertices of the pyramid ABCD, they will be able to view the entire Earth's surface: for each point p on the Earth, the closest satellite will be able to view the point p.

Now we prove that it is not possible to view the entire surface with 3 or fewer satellites regardless of the positions of the satellites. First, consider that any one satellite can view at most an area that approaches a hemisphere as the satellite is moved far from the Earth. This almost-hemisphere that it can view has a center point on its surface given by the intersection of the line from the satellite to the center of the Earth with the surface of the Earth. Now let X, Y, and Z be the positions of 3 satellites. Draw a plane through points X, Y, and Z (the plane is unique unless the points are collinear). Let line segment $\overline{NS}$ be the diameter of the sphere that is perpendicular to the plane. We claim that either N or S (or both) cannot be viewed from any of the satellites. To see this, note that the plane either contains the center of the sphere or is on one side or the other. If the plane contains the center of the sphere, then the centers of the almost-hemispheres for each satellite are on the equitorial circle that is the intersection of the plane with the surface of the sphere. In this case, none of the almost-hemisphere viewing areas of the satellites includes either N or S. If the plane is on either side of the center of the sphere, then the N or S which is on the opposite side will not be in the viewing area of any of the satellites since the almost-hemispheres are further from either N or S. Thus, 3 or fewer satellites cannot view the entire surface of the Earth at one time.

Challenge of the Week # 10 - April 11 to April 18: In the Jungle, you are in a small clearing with nine very large caves surrounding you which are filled with a practically unlimited number of gold coins.

In one cave the coins all weigh 1 ounce; in another cave the coins allweigh 2 ounces; in another cave the coins all weigh 3 ounces; and so on with the coins all weighing 9 ounces in the last cave.

You have a large scale which can be used to weigh coins. What is the least number of weighings that are needed in order to find out which cave contains which weight of coin? Explain how.

Complete solution submitted by Shane Reichart ($10 prize). Partial solutions submitted by Jamie Baker, Jenny Bauer, Anne Beil, Brent Furrow, Julia Smith, Symon Shmukler, Megan Sparks, and Sharon Tuttle. Solution: (due to Shane Reichart) The least number of weighings that are needed is just one!

Take one coin from the 1^{st} cave, 10 from the 2^{nd} cave, 100 from the 3^{rd} cave, etc. until you reach the 8^{th} cave (it is unnecessary to use coins from the 9^{th} cave because once all other weights are determined, the weight of the coins in this cave is the digit 1-9 that is not used). Weigh all of these 11,111,111 coins together. The resulting weight is an 8 digit number with the property that the units place digit is the weight of a coin in the 1^{st} cave, the 10's place digit is the weight of a coin in the 2^{nd} cave, etc. This is because, for example, there are 100 coins from the 3^{rd} cave, so the total weight of these coins is between 100 and 900 with the weight of one such coin equal to the digit in the 100's place.

Challenge of the Week # 11 - April 18 to April 25: You have 100 sticks such that no two have the same length. You would like to form a rectangle by laying all of the sticks end-to-end around the perimeter of the rectangle. You may break some of the sticks, but no stick may be broken more than once and you must use all pieces. Depending on the lengths of the sticks, sometimes you can form the rectangle without breaking any of them and sometimes you will have to break one or more sticks. What is the least number n such that no more than n breaks are required regardless of the lengths of the given sticks? Show that your answer can always be achieved and that it is the least such number.

No complete solutions were submitted ($20 prize next week). Partial solutions submitted by Jamie Baker, Anuj Bhandari, Julie Kleinschmidt, Stephanie Lewis, Shane Reichart, Julia Smith, and Megan Sparks. Other submissions from Jenny Bauer, Anne Beil, and Sharon Tuttle. Solution: Take any stick and break it in half. These two pieces will be used for two opposite sides of the rectangle. Line the remaining sticks up end-to-end and find the midpoint of this length. If the midpoint falls on a stick, break it at the point. Then use the sticks on either side of the midpoint for the remaining two opposite sides of the rectangle. Thus, no more than 2 breaks are ever necessary.

Now we will show that it is not possible to always get a rectangle with one break. Suppose one stick is 100 ft long, and the lengths in feet of the remaining 99 sticks equal the reciprocals of the first 99 primes. That is, $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{11}, \frac{1}{13}, \frac{1}{17}, \dots \frac{1}{523}$. The 100 ft long stick must be broken since the sum of the remaining lengths is less than 50 ft (they are all $\leq \frac{1}{2}$). The two pieces that it is broken into must be used on opposite sides of the rectangle since if not, the side opposite the longer of the two would have to be made up of the short pieces which do not equal 50 ft total. To form a rectangle with no other breaks, requires that the two sides which use the long pieces, possibly also use some of the small pieces, and then the remaining small pieces must be able to be divided into two groups that have the same total length for the remaining two opposite sides of the rectangle. Thus we must have $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} + \cdots + \frac{1}{p_m} = \frac{1}{q_1} + \frac{1}{q_2} + \frac{1}{q_3} + \cdots + \frac{1}{q_n}$ for some positive integers m and n where the p_i's and q_i's are all distinct primes. Multiplying by the least common denominator on both sides of the equation results in an equation such that p_1 is a factor of every term except the first term. This is a contradiction (since p_1 would then have to be a factor of the first term too), so for the given sticks, it is not possible to form a rectangle with just one break.

Challenge of the Week # 12 - April 25 to May 2: There is a 5x5 table filled with positive numbers. The product of the numbers in each row and in each column is 1. Also the product of the numbers in each 2x2 square is 2. Find all such tables and show that you have found them all.

No complete solutions were submitted ($30 prize for the first problem next Fall). Partial solutions submitted by Audra Frericks, Brent Furrow, Shane Reichart, and Sharon Tuttle.

Solution: Let the number in the ith row and the jth column be denoted by $a_{i,j}$. The 2x2 square given by $a_{1,1}$, $a_{1,2}$, $a_{2,1}$, and $a_{2,2}$ and the 2x2 square given by $a_{3,1}$, $a_{3,2}$, $a_{4,1}$, and $a_{4,2}$ both must have product 2. Also, since the product of numbers in each column is 1, the product of all of the numbers in the first two columns must be 1. The product of the numbers in the first two columns consists of the same numbers as those in the above two 2x2 squares except that $a_{5,1}$ and $a_{5,2}$ are also included. The product of the numbers in the two 2x2 squares is $2 \cdot 2 = 4$ so $a_{5,1} \cdot a_{5,2}$ must equal $\frac{1}{4}$. Similarly, by working with columns 3 and 4, $a_{5,3} \cdot a_{5,4} = \frac{1}{4}$. Thus, the product of the numbers in row 5 is $\frac{1}{4} \cdot \frac{1}{4} \cdot a_{5,5} = 1$. Thus, $a_{5,5} = 16$ and by symmetry, the four corners each equal 16. That is, $a_{1,1} = 16$, $a_{1,5} = 16$, $a_{5,1} = 16$, and $a_{5,5} = 16$.

Now using that $a_{5,1} \cdot a_{5,2} = 16 \cdot a_{5,2} = \frac{1}{4}$ we get that $a_{5,2} = \frac{1}{64}$. By symmetry $a_{5,4} = \frac{1}{64}$.

So the product of numbers in row 5 is $16 \cdot \frac{1}{64} \cdot a_{5,3} \cdot \frac{1}{64} \cdot 16 = 1$ which results in $a_{5,3} = 16$.

We now know the entire 5th row and by symmetry we have the outside boundary complete. To get $a_{2,2}$ look at the upper left 2x2 square. Since we know all but $a_{2,2}$ we find that it is 512. By symmetry $a_{2,4}$ is also 512. By looking at row 2 we then get that $a_{2,3}$ must be $\frac{1}{64}$. Again using symmetry, we know all but the center square. Using the center column we get that the center square must be 16. The solution is shown in the figure below.

16	1/64	16	1/64	16
1/64	512	1/64	512	1/64
16	1/64	16	1/64	16
1/64	512	1/64	512	1/64
16	1/64	16	1/64	16