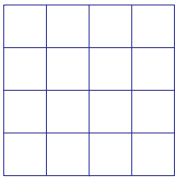
# Solution - Challenge of the Week

*Challenge of the Week # 3 - September 11 to September 18, 2009*

Is it possible to put the integers 1 through 16 in a $4 \times 4$ array of squares (as shown below) so that



(i) one integer is placed in each small square;

(ii) all 16 integers are used exactly once;

(iii) if you multiply the numbers in each of the four **rows** and add the four products, the result **is divisible by 5**; and

(iv) if you multiply the numbers in each of the four **columns** and add the four products, the result **is not divisible by 5**?

Justify answer by providing an example, if one exists, or by proving that no such example exists.

**This week's challenge was solved by Hilary Cloe, Joseph Leipert, Mollie Neff, William Petersen, Ashlee Sharp, and Cydnee Tucker. Other submissions came from Vicki Kane, Caitlin Lyman, and Brandon Martin.**

**Solution:** There are many different solutions. There are three multiples of 5 between 1 and 16, inclusive. Place one of them in the first row, first column; place one in the second row, second column; and one in the second row, third column. No matter how you place the other numbers 13 numbers, the product of the numbers in the fourth column will not be a multiple of 5. Thus, if you multiply the numbers in each column and then add the products, the result will not be divisible by 5.

But not all arrangements of the 13 other numbers will satisfy the first condition above. However, if you place 1,2,3,4 in the third row and 9,12,13,14 in the last, the product numbers in the third row plus the product of the numbers in the last row is

$$1 \cdot 2 \cdot 3 \cdot 4 + (10-1)(10+2)(10+3)(10+4).$$

which is a multiple of 10. The product of the numbers in the first and second rows will be multiples of 5. Hence the sum of the all the column products will be a multiple of 5. This gives the following example:

| | | | |
|---|---|---|---|
| 5 | 6 | 7 | 8 |
| 11 | 10 | 15 | 16 |
| 1 | 2 | 3 | 4 |
| 9 | 12 | 13 | 14 |