

Solution - Challenge of the Week

Challenge of the Week # 1 - August 28 to September 4, 2009

You have an integer with many digits. This integer is one more than a multiple of 9. One of the digits is removed and the remaining digits are rearranged. The new number is 8 more than a multiple of 9. What digit was removed? Justify your answer.

Complete solutions to this week's challenge were submitted by Jon Harter, Joseph Leipert, and Alex Meadows. Partial solutions were submitted by Jarod Barber, Hilary Cloe, Rebecca Dunning, Heather Gerrish, Charles James Jr., Gwen Maxedon, Ashlee Sharp, and Cydnee Tucker.

Solution: Suppose M is a positive integer whose decimal expansion is

$$a_n \cdot 10^n + \cdots + a_2 \cdot 10^2 + a_1 \cdot 10 + a_0,$$

where $a_0, a_1, a_2, \dots, a_n$ are integers between 0 and 9, inclusive. Then

$$M = [a_n(10^n - 1) + \cdots + a_2(10^2 - 1) + a_1(10 - 1)] + [a_n + \cdots + a_2 + a_1 + a_0].$$

Since 9 divides $10^m - 1$ for every positive integer m , 9 divides

$$a_n(10^n - 1) + \cdots + a_2(10^2 - 1) + a_1(10 - 1).$$

Hence, M is k more than a multiple of 9 if and only if the sum of the digits of M is k more than a multiple of 9. Therefore, the given problem is equivalent to:

You have an integer. The sum of the digits of this number is one more than a multiple of 9. One of the digits is removed and now the sum is 8 more than a multiple of 9. What digit was removed?

From this equivalent formulation it is clear that the digit that was removed was 2.