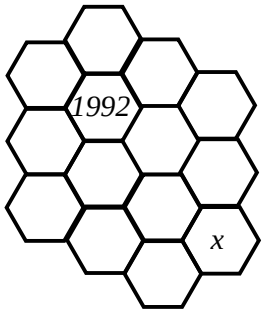
The Fifth Eastern Illinois University
Undergraduate Problem Solving Competition
1992 - 1993

1. *Find all positive integers n and k such that n^n has k digits and k^k has n digits.*

2. *Hexagons cover the xy-plane as in the following figure. Each hexagon contains a positive integer which is the average of the integers in the six hexagons surrounding it. One hexagon contains 1992. What is the value of x in the figure.*



3. *Two people start with the set of integers $\{1, 2, 3, \ldots, 100, 101\}$. They take turns removing 9 integers from this set. After 11 turns there will be two integers remaining, say a and b with $a > b$. The person with the first turn will receive $a - b$ dollars from the person with the second turn. What is the optimal strategy for each person and what is the payoff with these strategies?*

4. *Given a point P on side AB of triangle ABC, construct (with straight-edge and compass) a line through P which divides $\triangle ABC$ into two halves.*

5. *a) If you draw 9 line segments without lifting your pencil and end at the starting point, so that no three segments meet at the same point, what is the maximum number of intersections? Prove your answer is possible and is a maximum.*

   b) What about 10 line segments?

6. *Let a, b, c, d be positive real numbers. Define a sequence $\{s_i\}$, by $s_1 = (a, b, c, d)$ and, if $s_n = (w, x, y, z)$, $n \geq 1$, then $s_{n+1} = (w \cdot x, x \cdot y, y \cdot z, z \cdot w)$. Show that $s_n = s_m$, for some $n \neq m$, if and only if $s_2 = (1, 1, 1, 1)$.*