Problem 10

In a century, it is possible to perform $3.1536 \cdot 10^{25}$ operations. Thus it will take

$$\frac{10^{10^{100}}}{3.1536 \cdot 10^{25}} = \frac{10 \cdot 10^{10^{100}-1}}{3.1536 \cdot 10^{25}} = 3.171 \cdot 10^{10^{100}-26}$$

centuries to perform a googolplex opertions.

For extra credit: There are $10^{100} - 25$ characters in $3.171 \cdot 10^{10^{100}-26}$. At a rate of $3.1536 \cdot 10^{12}$ characters per century, it would take

$$\frac{10^{100} - 25}{3.1536 \cdot 10^{12}} = \frac{10 \cdot 10^{99}}{3.1536 \cdot 10^{12}} - \frac{25}{3.1536 \cdot 10^{12}} = 3.171 \cdot 10^{87}$$

centuries. (The last "equality"holds because the number that is being subtracted is approximately $8 \cdot 10^{-12}$.)

Problem 11b

We want to find $\{a_n\}$ and $\{c_{-n}\}$ such that

$$\frac{2009}{11} = \sum_{n=0} a_n \cdot 60^n + \sum_{n=1} c_{-n} \cdot 60^{-n}.$$

This can be accomplished using the following two algorithms,

Algorithm to find $\{a_n\}$.

Input: N, a real number
Output: $a_0, a_1, \ldots$ such that $\lfloor N \rfloor = a_0 + a_1 \cdot 60 + a_2 \cdot 60^2 + \cdots$.
$N \leftarrow \lfloor N \rfloor$
`Repeat until` $N = 0$.
    `Output` N mod 60. *(that is, $N - \lfloor N/60 \rfloor \cdot 60$)*
    $N \leftarrow \lfloor N/60 \rfloor$

Algorithm to find $\{c_{-n}\}$.

Input: N, a real number
Output: $c_{-1}, c_{-2}, \ldots$ such that $\{N\} = c_{-1}60^{-1} + c_{-2}60^{-2} + \cdots$.
$N \leftarrow \{N\}$
`Repeat until tired or` $N = 0$
    `Output` $\lfloor 60N \rfloor$.
    $N \leftarrow \{60n\}$.

Finding $\{a_n\}$ for $\frac{2009}{11} = 182\frac{7}{11}$ with base 60

Number of times through the loop	Value of N at the beginning of the loop	Output	Value of N at the end of the loop
1	280	280 mod 60 = 2	$\lfloor 280/60 \rfloor = 3$
2	3	3 mod 60 = 3	$\lfloor 3/60 \rfloor = 0$

Thus, $182 = 2 + 3 \cdot 60$ or $(3, 2)$.

Finding $\{c_{-n}\}$ for $\frac{7}{11}$ with base 60

Number of times through the loop	Value of N at the beginning of the loop	Output	Value of N at the end of the loop
1	$7/11$	$\lfloor 60(7/11) \rfloor = 38$	$\{60(7/11)\} = 2/11$
2	$2/11$	$\lfloor 60(2/11) \rfloor = 10$	$\{60(2/11)\} = 10/11$
3	$10/11$	$\lfloor 60(10/11) \rfloor = 54$	$\{60(10/11)\} = 6/11$
4	$6/11$	$\lfloor 60(6/11) \rfloor = 32$	$\{60(6/11)\} = 8/11$
5	$8/11$	$\lfloor 60(8/11) \rfloor = 43$	$\{60(8/11)\} = 7/11$
6	$7/11$	The pattern repeats!	

Thus,

$$\frac{7}{11} = (0; \overline{38, 10, 54, 32, 43})$$

and

$$\frac{2009}{11} = (3, 2; \overline{38, 10, 54, 32, 43})$$