Mathematics 4900 Mathematical Induction

Four "proofs" by mathematical induction.

Prove: $2 + 5 + \cdots + (3n-1) = \frac{n}{2}(3n+1)$**.**

"Proof 1"

$2 = \frac{1}{2}(4)$.

$k+1 = 2 + 4 + \cdots + (3(k+1)-1) = \frac{k}{2}(3k+1) + (3(k+1)-1) = \frac{k+1}{2}(3k+4)$.

Comments: There is no structure to this proof. The notation is very confused. In the second line, an equation,

$$2 + 4 + \cdots + (3(k+1)-1) = \frac{k}{2}(3k+1) + (3(k+1)-1) = \frac{k+1}{2}(3k+4),$$

seems to be confused with an expression, $k+1$. This is "proof" has no value.

"Proof 2"

1) Check that $P(1)$ is true: $(3(1)-1) = \frac{1(3+1)}{2} = 2$.

2) Since $P(k)$ is true, check for $P(k+1)$.

$$3(k+1) - 1 = 3k + 2$$

$$\frac{k+1}{2}(3(k+1)-1) \quad \frac{(k+1)(3k+4)}{2}.$$

So $P(k+1)$ true.

3) By PMI, 1) and 2) $P(n)$ is true.

Comments: There is some structure here, but after that the logic falls apart. What is $P(n)$? Is it an aardvark? The statement "Since $P(k)$ is true" is meaningless at this point. What is meant "Assume by induction, that $P(k)$ is true". Probably a D because the basic structure of induction is illustrated.

"Proof 3"

$$\begin{aligned}
2 &= \frac{n}{2}(3n+1) \\
2 + 5 + \cdots (3(k+1)-1) &= 2 + 5 + \cdots 3k - 1 + 3k + 2 \\
&= \frac{k}{2}(3k+1) + 3k + 2 \\
&= \frac{(k)(3k+1) + 6(k+4))}{2} \\
&= \frac{3k^2 + 7k + 4}{2} \\
&= \frac{(k+1)(3k+4)}{2} \\
&= \frac{k+1}{2}(3(k+1)+4)
\end{aligned}$$

Comments: There is no structure here. All of the algebraic steps are here, but there is no explanation. Passing, but probably only a C. Unfortunately, this is to a proof as "light hailed watched glare proof banner free brave" is to the national anthem.

Proof: Let $P(n)$ be the statement,

$$2 + 5 + \cdots + (3n-1) = \frac{n}{2}(3n+1),$$

where n is a positive integer.

When $n = 1$, the left hand side $P(1)$ is 2 while the right hand side is $\frac{1}{2}(3 \cdot 1 + 1) = 2$. Hence $P(1)$ is true.

Assume, by way of induction, that $P(k)$ is true for some positive integer k. That is, assume

$$2 + 5 + \cdots + (3k - 1) = \frac{k}{2}(3k + 1).$$

Adding $3(k + 1) - 1$ to both sides of this equality gives

$$2 + 5 \cdots + (3k - 1) + (3(k + 1) - 1) = \frac{k}{2}(3k + 1) + (3(k + 1) - 1).$$

Simplifying the right hand side and finding a common denominator yields

$$\begin{aligned} 2 + 5 + \cdots + (3k - 1) + (3(k + 1) - 1) &= \frac{k}{2}(3k + 1) + (3k + 2) \\ &= \frac{k(3k + 1) + 2(3k + 2)}{2} \\ &= \frac{3k^2 + 7k + 4}{2} \end{aligned}$$

Since $3k^2 + 7k + 4 = (k + 1)(3k + 4)$, we have

$$2 + 5 + \cdots + (3k - 1) + (3(k + 1) - 1) = \frac{(k + 1)(3k + 4)}{2}.$$

This equation is easily rewritten as

$$2 + 5 + \cdots + (3(k + 1) - 1) = \frac{k + 1}{2}(3(k + 1) + 1).$$

This is the statement $P(k + 1)$. Hence, $P(k + 1)$ is true, provided that $P(k)$ is true.

The preceeding paragraphs and the Principle of Mathematical Induction imply that $P(n)$ is true for all positive integers n.

Comments: The proof begins with a statement which defines $P(n)$. Without this statement, the "reader" of the proof will have no idea of what $P(n)$ is. In the proof, the three parts of an induction proof are clearly indicated.

- A verification of a basis case.
- The explanation as to why the truth $P(k)$ for an arbitrary, but specific, value of k implies the truth of $P(k + 1)$.
- The statement that first two parts of the induction proof, together with the Principle of Mathematical induction, imply that the statement under consideration is true for all relevant values of n.

Finally, an explanation is given for most algebraic steps.

Note: There are many ways to write a "poor" proof. There are many, but not quite as many, ways to write a "good" proof.