Persistently Foliar Knots, II

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Joint work with Rachel Roberts, WUSTL

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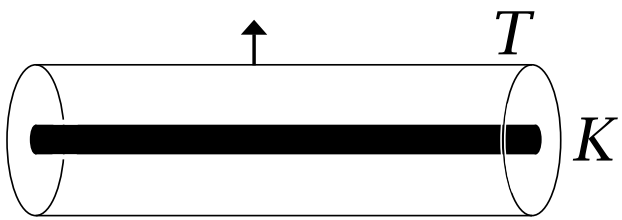
Work in the Knot *Exterior*

Now:

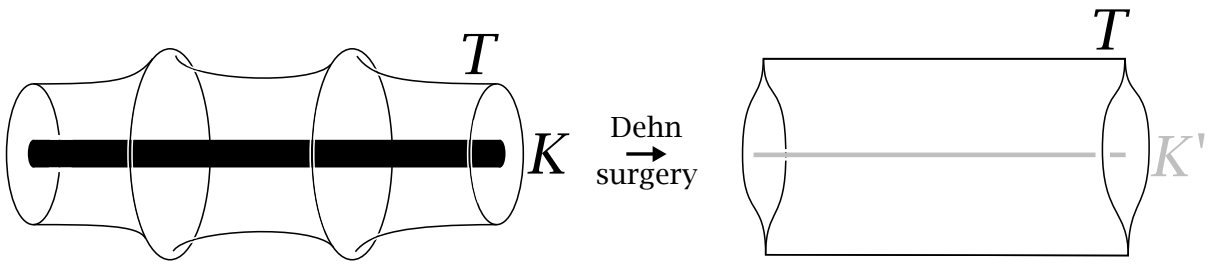
- Work in the knot *exterior:* $S^3 \setminus K$
- Introduce a "tube" around K: $T = \partial N(K) \subset S^3 \setminus K$
- T is part of the spine.
- Convention: Outward normal to T points into knot complement, out of $N(K)$.



Meridional Cusps → Persistence

Goal:

- Build spine having meridional intersections with T.
- Smooth to branched surface Σ with even (> 0) number of meridional branch curves with outward sink direction on T.
- After any rational Dehn surgery, these yield an even number of longitudinal sutures, so a meridional disk fully decomposes $N(K')$ (as a taut sutured manifold).

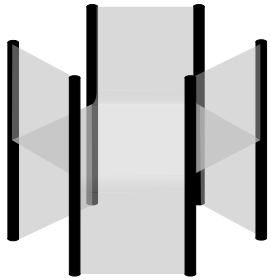


Meridional Cusps $\rightarrow$ Persistence (continued)

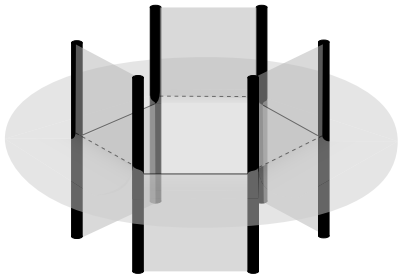
- Thus, as long as the other components of $\overline{N(\Sigma)^c}$ are taut sutured manifolds, we obtain a taut co-orientable foliation in every manifold produced by (non-trivial) surgery.
- This is what we mean by *persistence*.
- Antecedent: "Swallow-follow" closed (branched) surface. (Menasco; Oertel)

Method 1: Decomposition by Spheres & Spanning Surfaces

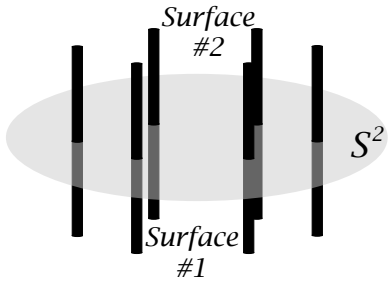
- Decompose K into tangles along transverse spheres.
- Decompose further along spanning surfaces for the tangles.
- Similar to Murasugi sum, but surfaces on each side need not match.



Murasugi sum



Branched Murasugi sum



Generalized decomposition along spheres and spanning surfaces

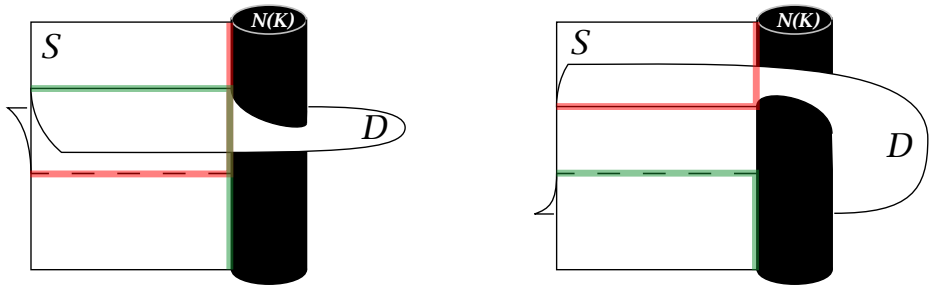
- With suitable choices, we obtain persistence, and every component of $N(\Sigma)^c$ is a taut sutured manifold.

Application of Method 1

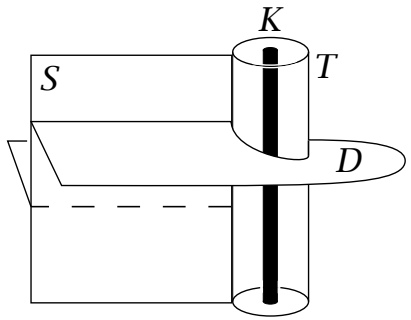
- Method 1 works well for Montesinos knots, since they decompose into rational tangles.
- Method 1 shows all Montesinos knots to be persistently foliar except for some "small" pretzel knots.

Method 2: Decomposition of a Spanning Surface

May be viewed as a generalization of Gabai's theory:



Sutured manifold decomposition
of a Seifert surface



Generalized surface decomposition
of a spanning surface

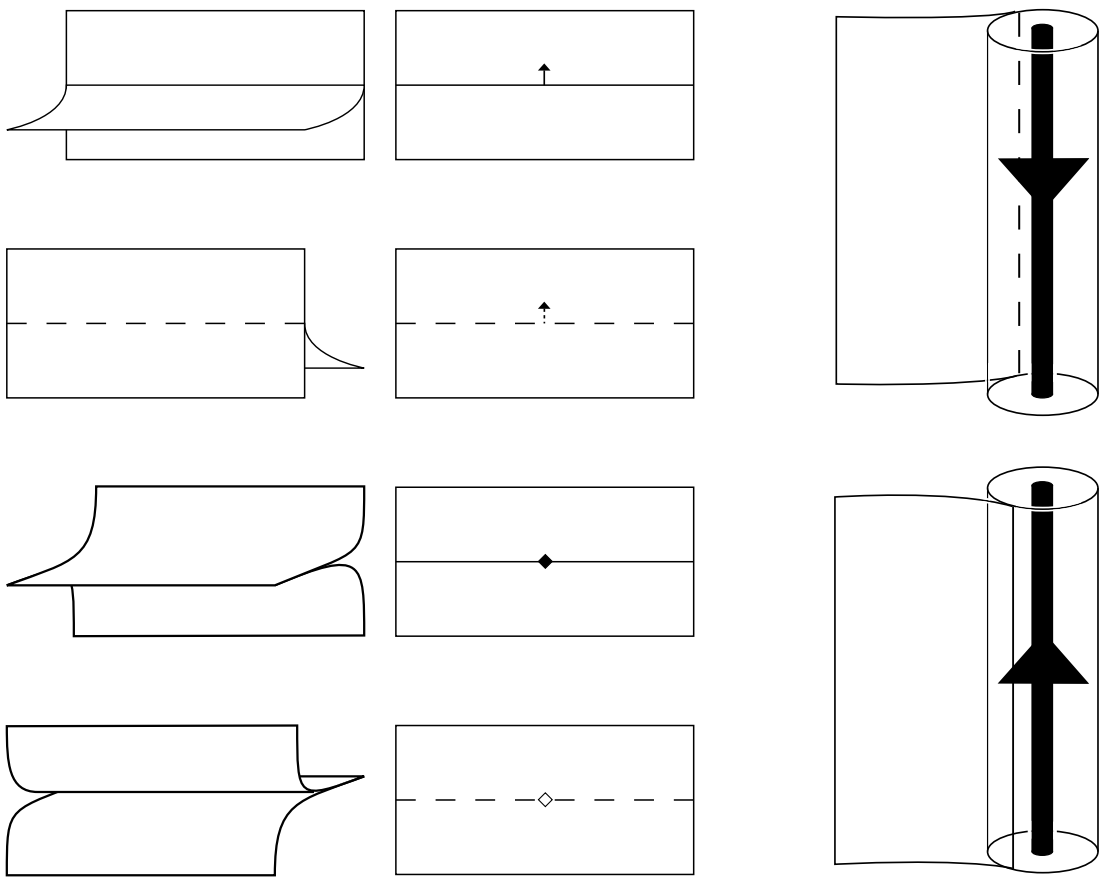
Some Differences; Application

Generalized decomposition of a spanning surface provides much greater flexibility:

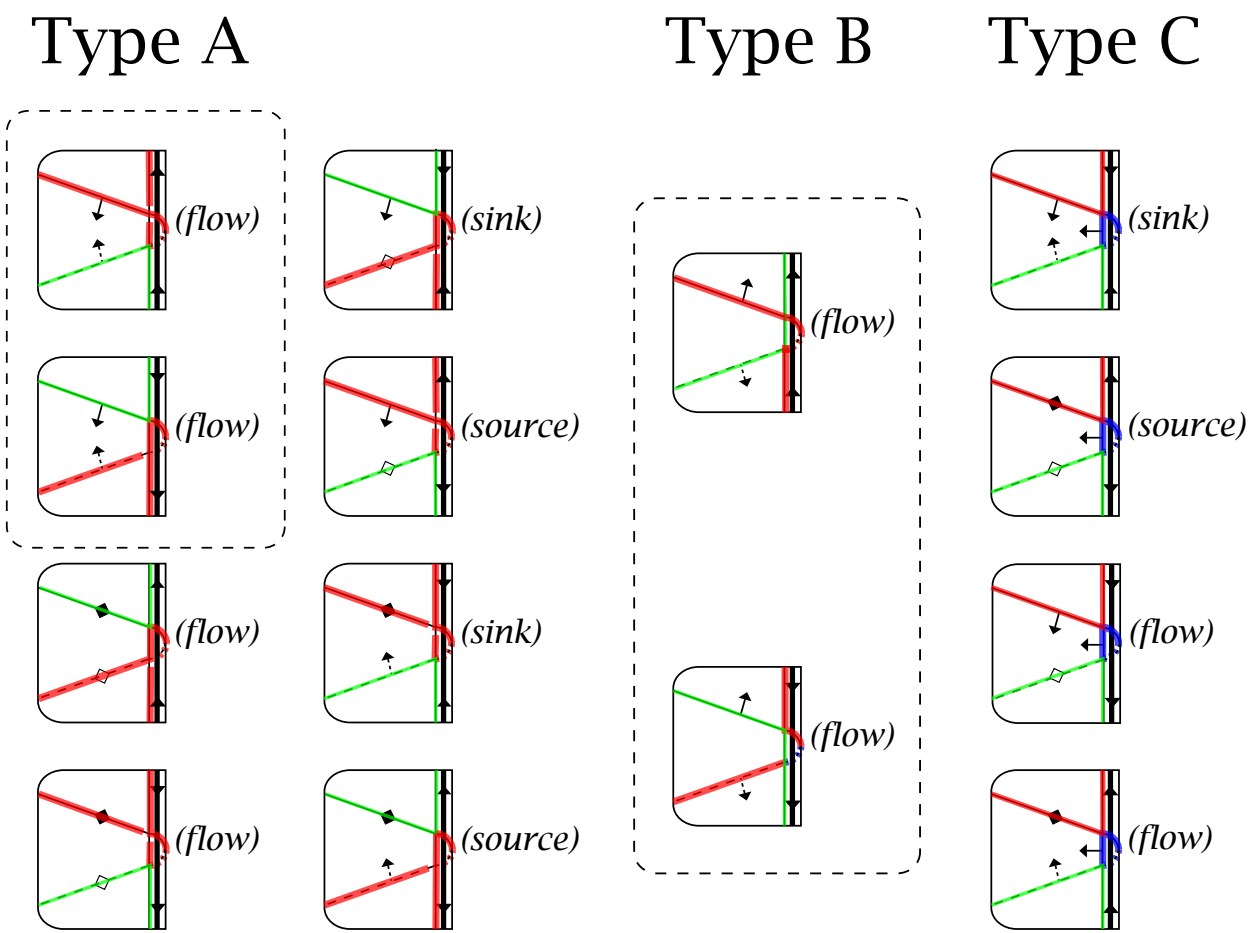
- Persistence.
- Initial spanning surface need not be orientable!
- Boundary of decomposing surface can cross over T from one side of S to the other an odd number of times!

Method 2 shows all non-torus alternating knots and all remaining pretzel knots that are not L-space knots to be persistently foliar.

Notation Conventions



Local Models



The $(-2, 5, 5)$ Pretzel Knot is Persistently Foliar

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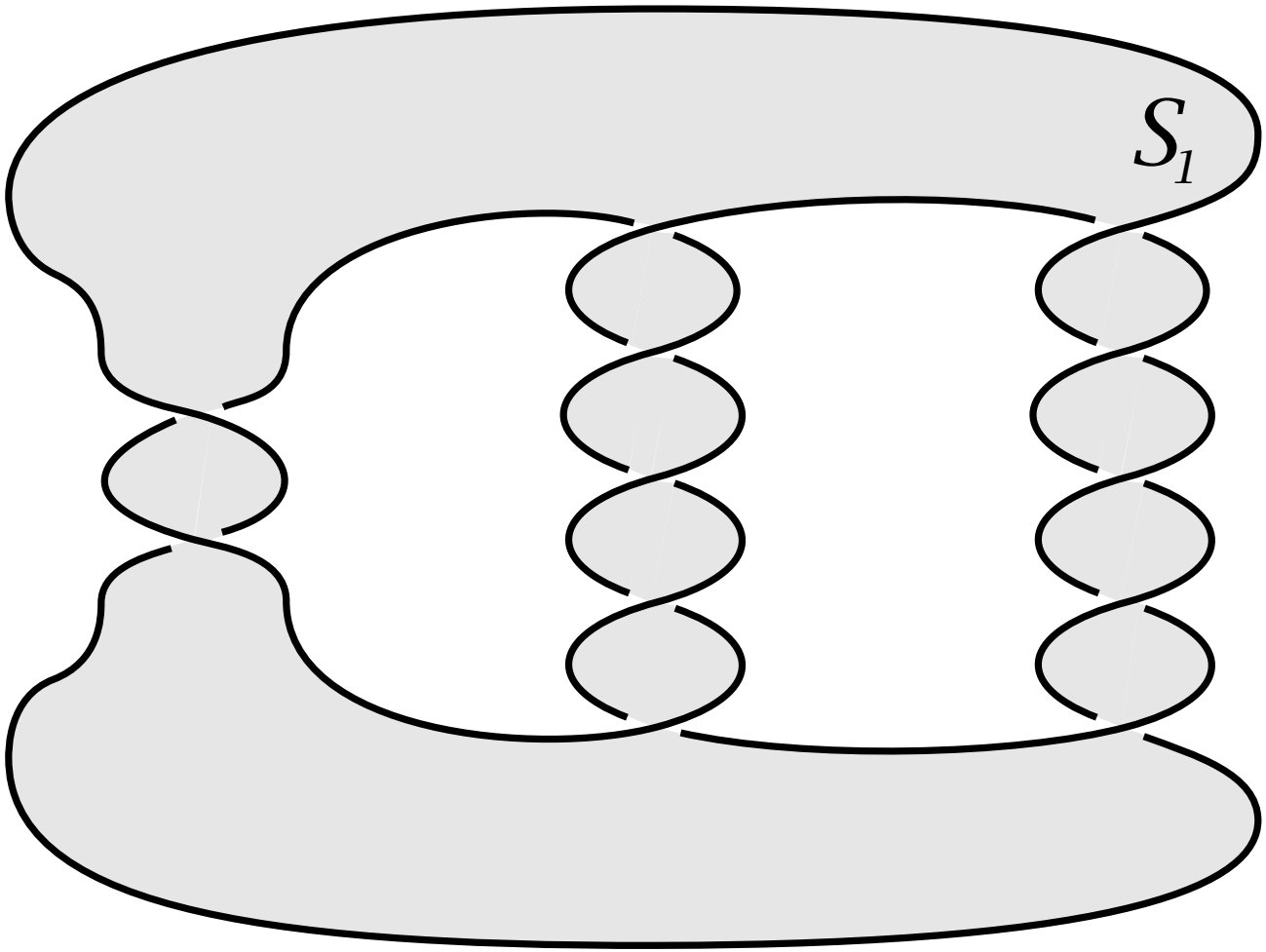

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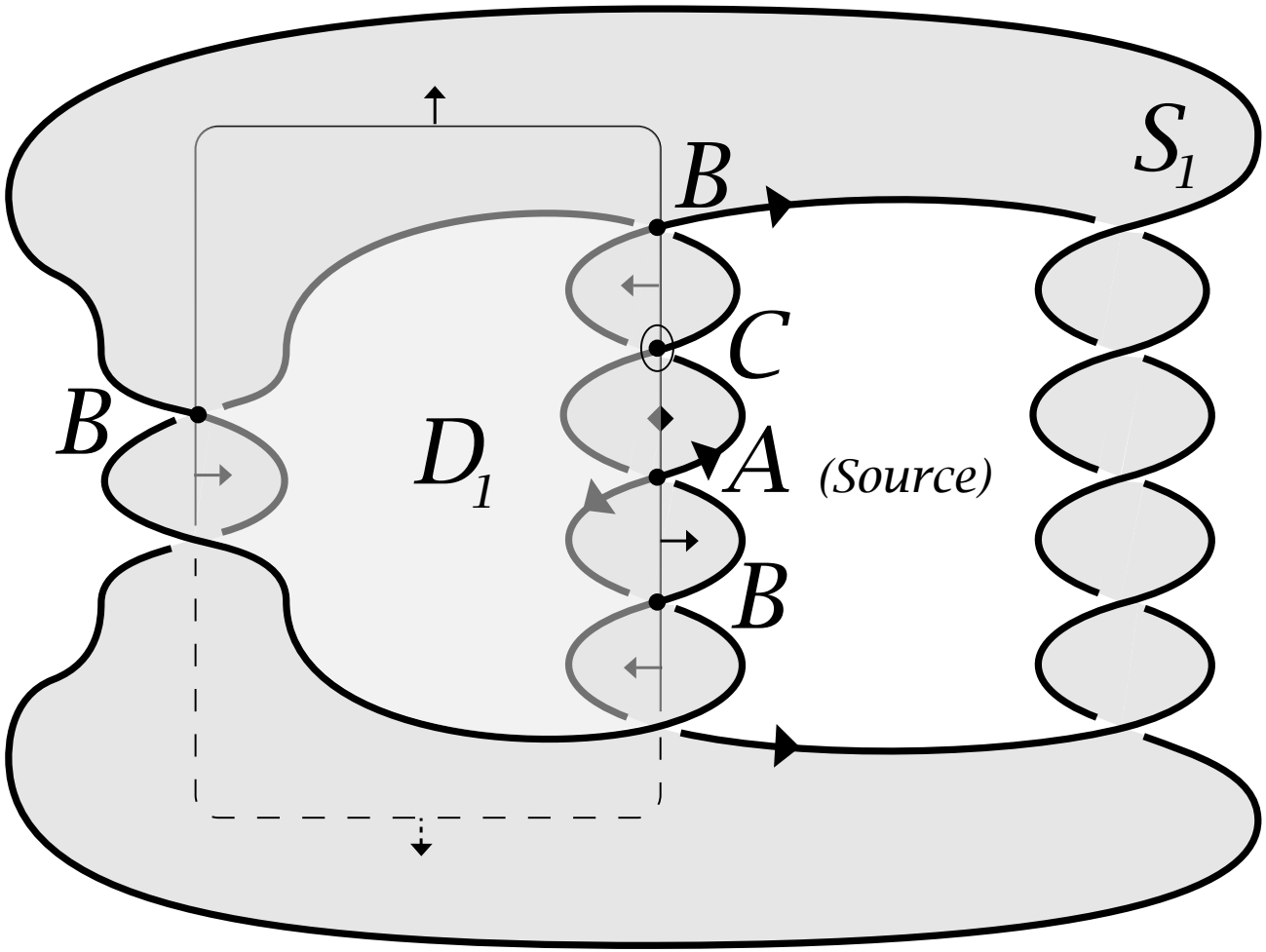

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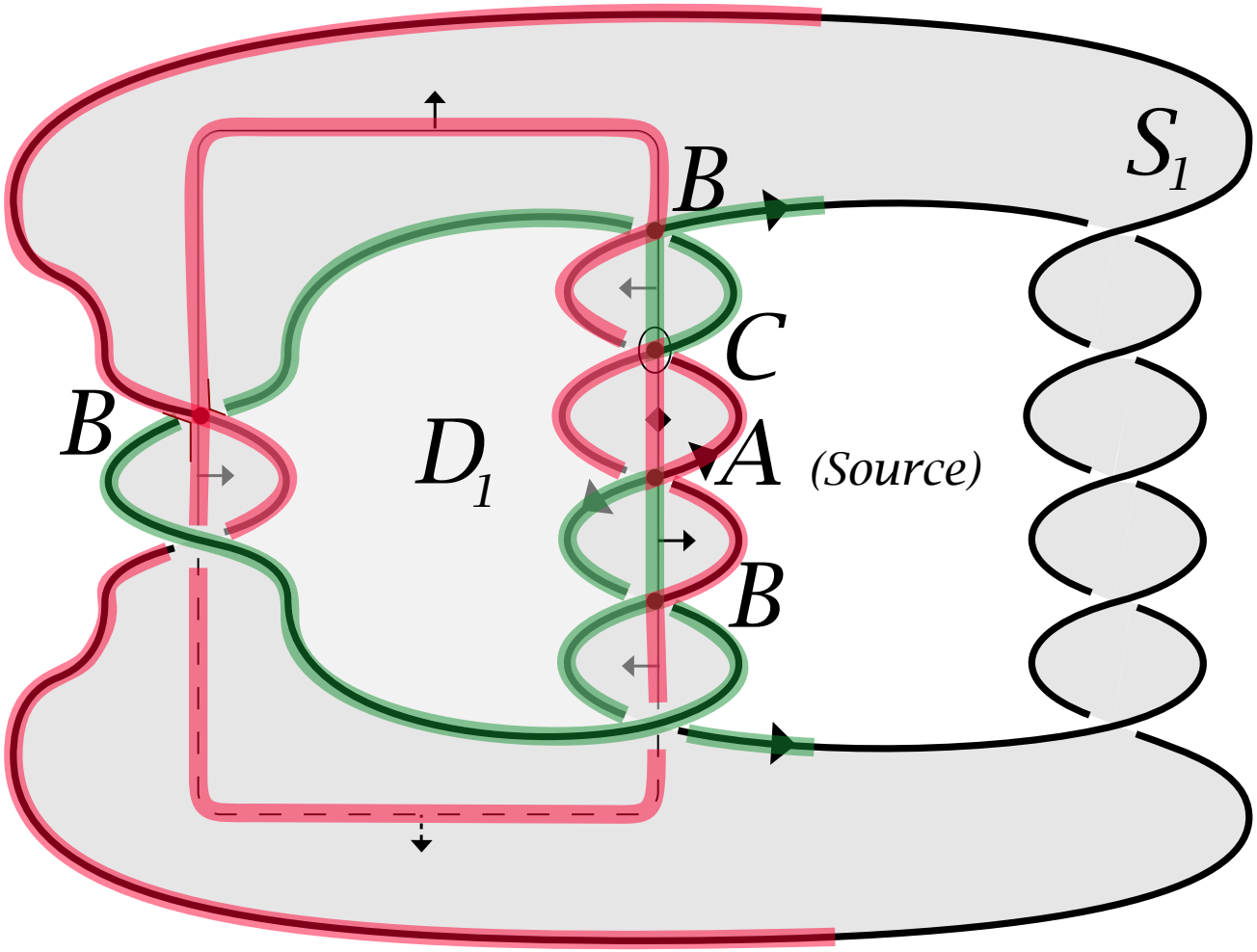

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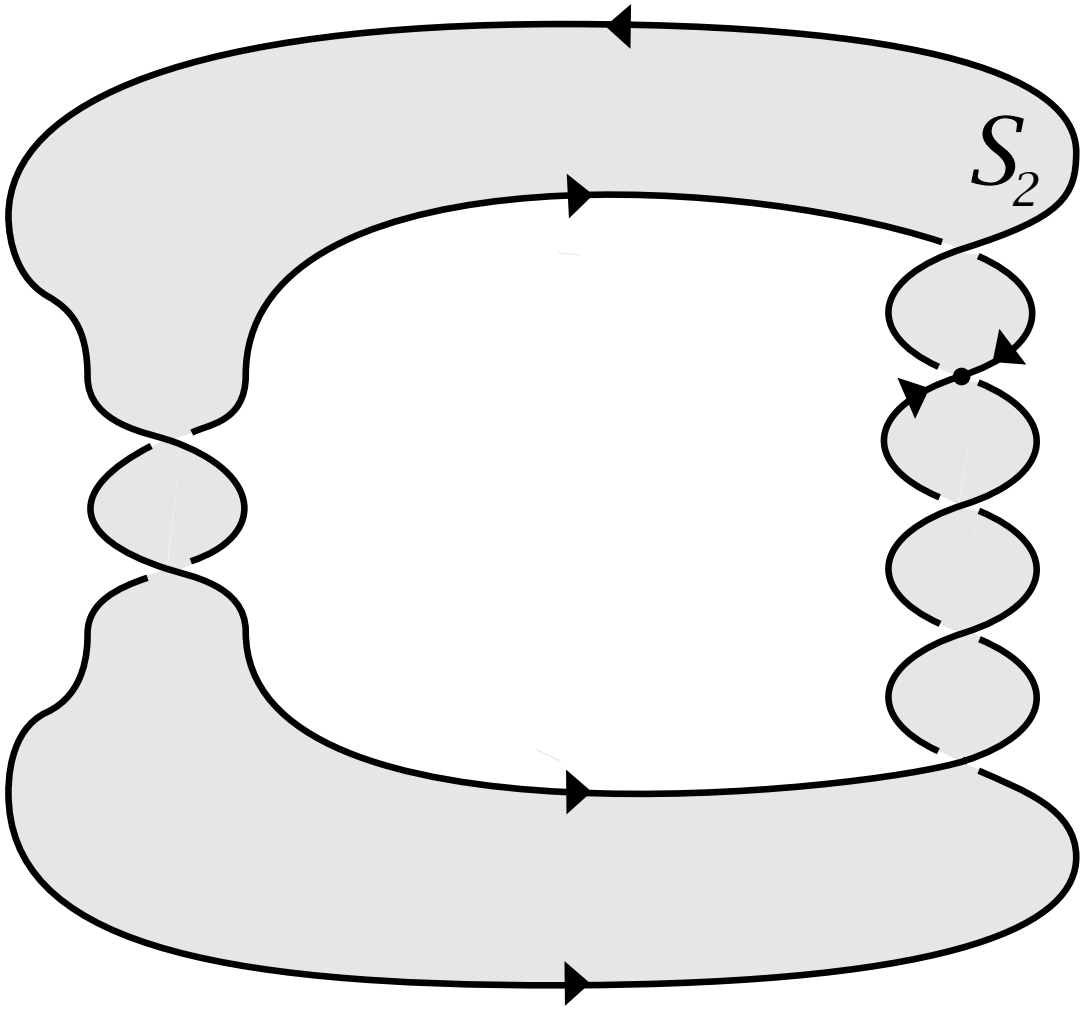

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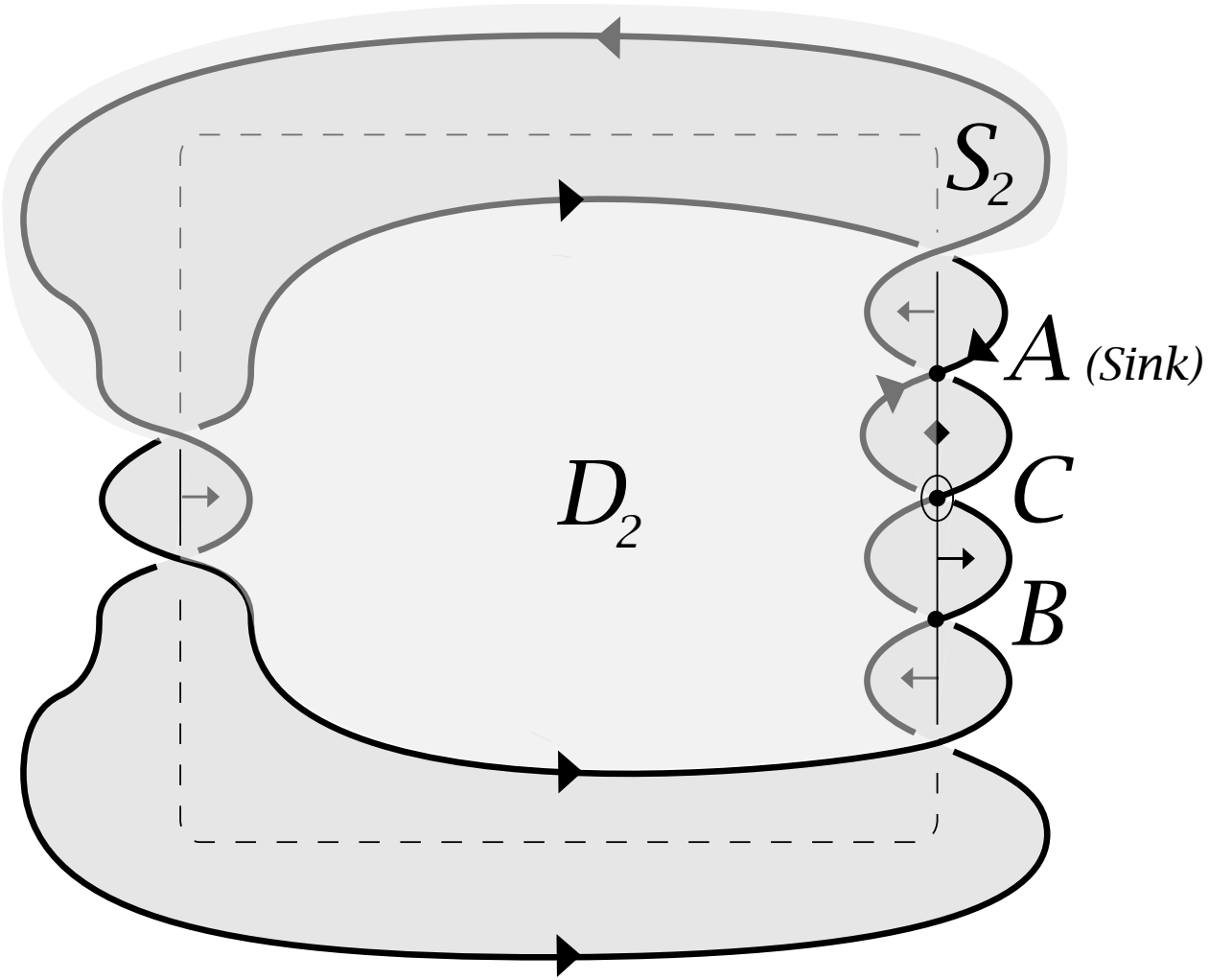

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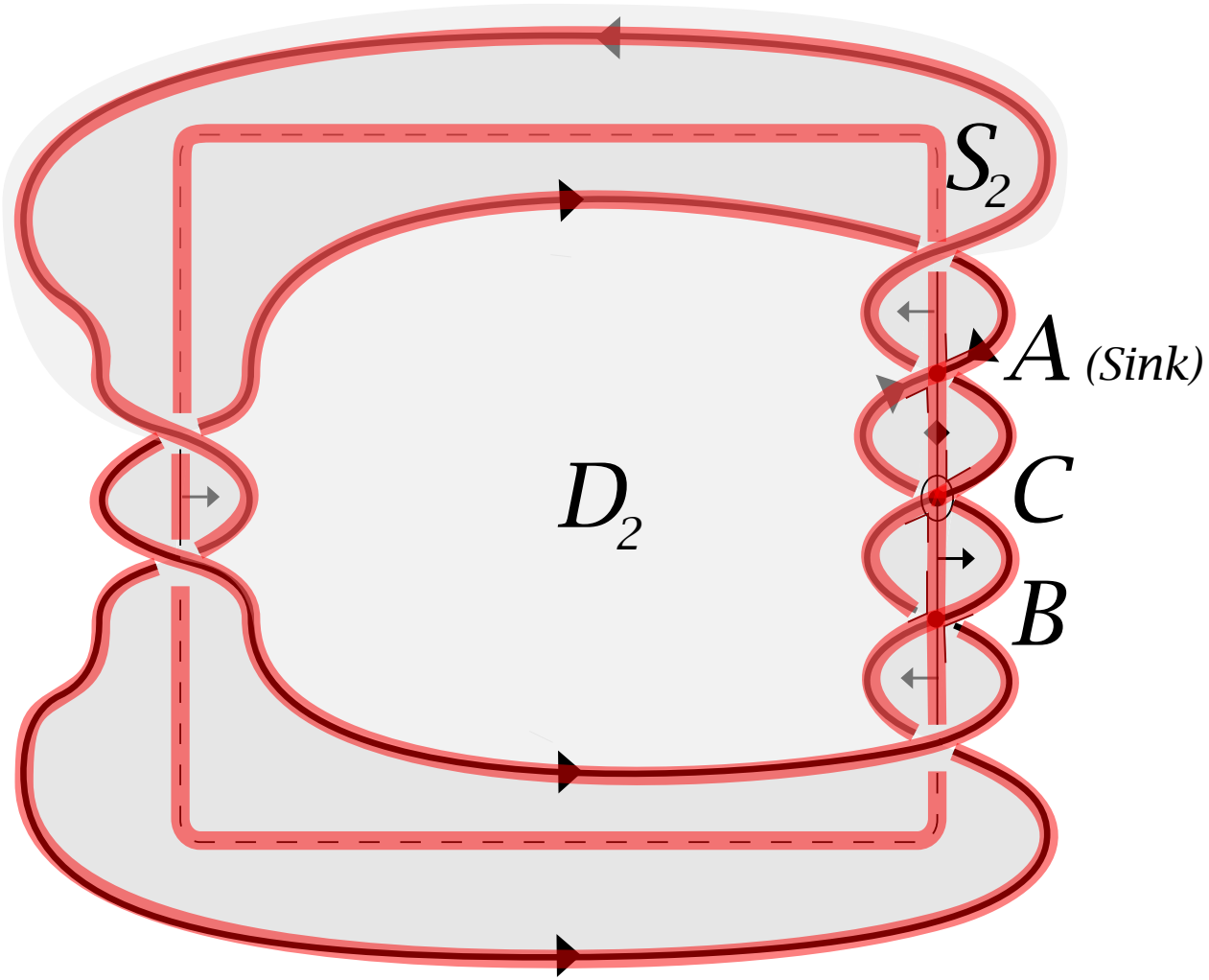

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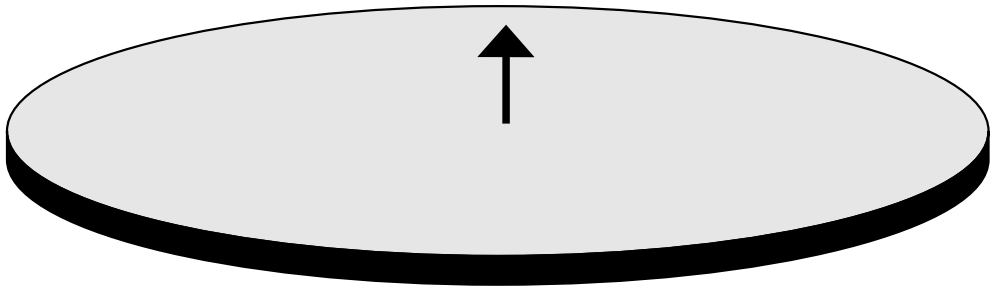

Questions?

Thank you!