MAT 5220, Topology **Name:** ______________________________
Midterm Exam - In-class portion. February 27, 2019

Let X and Y be topological spaces; let A, B, and A_α, $\alpha \in \mathcal{J}$, be subsets of X. Let $\mathrm{Int}A$ denote the interior of A, and let $\overline{A}$ denote the closure of A. You may use any of the equivalent definitions of interior and closure that you wish in proving the following:

1. (a) $\mathrm{Int}(A \cap B) = \mathrm{Int}(A) \cap \mathrm{Int}(B)$.

   (b) $\mathrm{Int}(\bigcap A_\alpha) \subseteq \bigcap \mathrm{Int}A_\alpha$.

   (c) Provide a counterexample showing that equality need not hold in the previous statement.

2. (a) $\mathrm{Int}(\bigcup A_\alpha) \supseteq \bigcup \mathrm{Int} A_\alpha$.

(b) Provide a counterexample showing that equality need not hold in the previous statement. (Hint: a union of two sets suffices; equality does not hold for finite unions, either.)

3. (a) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

(b) $\overline{\bigcup A_\alpha} \supseteq \bigcup \overline{A_\alpha}$

(c) Provide a counterexample showing that equality need not hold in the previous statement.

4. (a) $\overline{\bigcap A_\alpha} \subseteq \bigcap \overline{A_\alpha}$

(b) Provide a counterexample showing that equality need not hold in the previous statement. (Hint: an intersection of two sets suffices; equality does not hold for finite intersections, either.)

5. Let $\mathcal{B}$ be a basis for a topology on Y. Prove that $f : X \to Y$ is continuous if and only if the pre-image $f^{-1}(B)$ of every basic open set $B \in \mathcal{B}$ is open.