MAT 2550: Quiz 1 **Name:** ____________________

February 18, 2019

1. Find all real solutions to the following system of linear equations. (There are infinitely many!) You may express your solution in any form you like, as long as it is correct.

$$\begin{array}{lllll} 2x & & +z & +w & = 5 \\ & y & & -w & = -1 \\ 3x & & -z & -w & = 0 \\ 4x & +y & +2z & +w & = 9 \end{array}$$

2. Compute the following:

(a) $3\begin{pmatrix}1\\0\\0\end{pmatrix}+5\begin{pmatrix}0\\1\\0\end{pmatrix}-2\begin{pmatrix}0\\0\\1\end{pmatrix}=$

(b) $\begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}=$

(c) $\begin{pmatrix}1 & 0 & -1\\-1 & 1 & 0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}=$

(d) $\begin{pmatrix}1 & 0 & -1\\-1 & 1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\2 & 1\\1 & 3\end{pmatrix}=$

3. Let $T:\mathbb{R}^2\to\mathbb{R}^3$ be the linear map defined by $T\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}2\\1\\0\end{pmatrix}$ and $T\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1\\2\end{pmatrix}$. What is $T\begin{pmatrix}2\\5\end{pmatrix}$?

4. Let $T : V \to W$ be a linear map. Prove that $N = \{v \in V : Tv = \vec{0}\}$ is a subspace of V. (It is called the *Null Space* of T. Hint: It suffices to show that, if $v_1 \in N$ and $v_2 \in N$, then $\alpha v_1 + \beta v_2 \in N$. So let $v_1, v_2 \in V$ satisfy $Tv_1 = Tv_2 = \vec{0}$, let α, β be scalars, and show that $T(\alpha v_1 + \beta v_2) = \vec{0}$.)

5. Suppose that, for the set of vectors $\{v_1, v_2, v_3, \ldots, v_n\}$ in a vector space V, there exist scalars $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ and $\beta_1, \beta_2, \beta_3, \ldots, \beta_n$ such that $\alpha_1 \neq \beta_1$ and $\sum_{i=1}^{n} \alpha_i v_i = \sum_{i=1}^{n} \beta_i v_i$. Show that the vectors $v_1, v_2, v_3, \ldots,$ and v_n are linearly dependent (that is, *not* linearly independent). [Hint: subtract!]

Extra Credit!:

- Prove that for any linear map $T : V \to W$, $T(\vec{0}) = \vec{0}$. (Hint: Let $v \in V$ be any vector, and use the fact that $\vec{0} = 0v$, as proven in class on Wednesday; similarly, of course, $0w = \vec{0}$ for any $w \in W$. Alternatively, apply T to both sides of the identity $\vec{0} + \vec{0} = \vec{0}$.)