MAT 2550: Midterm Exam **Name:** ____________________
March 9, 2020.

Be sure to show your reasoning and process on all problems! Each numbered problem is worth 20 points. Feel free to use the last page, which is blank, for extra space.

1. Solve the following system of equations. (Hint: there are infinitely many solutions; adding twice the first row to the third is an efficient first steip.)

$$\begin{array}{cccccc} -x & +y & +4z & = & -1 \\ x & -y & +2z & = & 1 \\ 2x & -2y & -8z & = & 2 \end{array}$$

2. Compute the following:

   Note: all computations are possible.

   (a) $3\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}1\\1\end{pmatrix} =$

   (b) $\begin{pmatrix}1 & 0\\-1 & 1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} =$

   (c) $\begin{pmatrix}1 & 0\\-1 & 1\end{pmatrix}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} =$

   (d) $\begin{pmatrix}1 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix} =$

3. Let $\lambda : V \to W$ be a linear map. The *null space* of λ is defined by $\text{Null}\lambda = \{v \in V : \lambda v = \vec{0}\}$. Prove that $\text{Null}\lambda$ is a subspace of V.

4. Prove that the polynomials 1, $1+x$, and $1+x+x^2$ are linearly independent.

5. Suppose V and W are vector spaces, $\{v_1, v_2, v_3\}$ is an ordered basis for V, and $\{w_1, w_2\}$ is an ordered basis for W. Suppose $\lambda : V \to W$ is the linear map determined by the following values:

   - $\lambda v_1 = w_1 - w_2$
   - $\lambda v_2 = 2w_1$
   - $\lambda v_3 = 3w_2$.

   Provide the matrix for λ in terms of the given bases.