MAT 1400: Study Guide for the Final Exam

April 25, 2019

The final exam will be comprehensive, so you need to focus your studying on what is most important. You should also focus on thoroughly understanding the basic ideas behind the topics we covered, since if you understand those you can apply them to a wide variety of problems, and you can work with the confidence of knowing why what you are doing is correct. I've notice that some of you focus on the sections to be covered. Of course, you do need to know the topics that will be covered, but memorizing techniques section by section is not the way to study for a college course or to learn mathematics well. Definitely practice doing problems – absolutely, but also focus on the *ideas* covered by the exam.

Another bit of advice is that you need to be over-prepared. Study well enough that you can do problems that are harder than the ones I am likely to ask on the exam. If you do that, most the problems that actually appear on the exam will seem easy to moderately difficult, and you will be able to do them, even if you are nervous.

You will not be allowed to use calculators, but the arithmetic involved will be very easy. Calculators are useful tools, but they undermine your mathematical progress if you are excessively dependent on them. Before you use a tool, you need to understand how it does what it does.

Since there are many problems in the text you can study from, I have only provided a few examples below. The purpose of this study guide is to help you prioritize which topics to study. Make sure you thoroughly understand how to do all of the following:

- Solve equations and inequalities, including those with complex roots, such as $x^2 - 5x+6 = 0$, $x^2-3x+6 = 0$, $x^2-9 = 0$, $x^2+9 = 0$, $3(x-1)^{-\frac{1}{3}}+(x-1)^{\frac{2}{3}}(x-5) = 0$, $|x-5| < 2$, $|x-5| \geq 2$, $(x-1)(x-2)(x-3)^{-1} < 0$, etc.

- Write the equation of a circle and determine if a point is inside, on, or outside a circle.

- Write the equation of a line passing through two points, a line passing through a point and parallel to a given line, or a line passing through a point and perpendicular to a given line, and determine if a given point lies on a given line.

- Solve applied problems that require you to find unknown quantities by setting up an equation that models the problem. For example, a rectangular box has a square base, and its height is twice its width. If the box volume of the box is 16 cubic meters, what are its dimensions.

- Graph any function obtained by transforming one of the basic functions on page 166 of the text.
- In particular, graph any quadratic function using the various techniques that are applicable, such as putting the function in the standard form $f(x) = a(x-b)^2 + c$ or factoring it (or both), and accurately determine its intercepts and vertex.
- Determine the intervals on which a function is increasing or decreasing, and identify any maximum or minimum values.
- Determine the domain (meaning the largest possible domain) of a function, including those obtained by combining other functions using the basic operations of addition, subtraction, multiplication, division, and composition.
- Write the formula for a function obtained by combining given functions using the basic operations of addition, subtraction, multiplication, division, and composition.
- Write an improper rational function as the sum of a polynomial function and a proper rational function using long division of polynomials; in this context, proper means that the degree of the numerator is less than the degree of the denominator. This process is exactly analogous to writing an improper fraction as the sum a whole number and a proper fraction:

  Since

$$\begin{array}{r} 3 \\ 2\overline{)\,7} \\ \underline{6} \\ 1 \end{array}$$

  we find that $7 = 3 \cdot 2 + 1$ and, equivalently, dividing both sides by 2, we have

$$\frac{7}{2} = 3 + \frac{1}{2}.$$

  Since

$$\begin{array}{r} x - 1 \\ x^2 + x - 1\overline{)\;x^3 \quad + 3x + 5} \\ \underline{-x^3 - x^2 \;\; + x} \\ -x^2 + 4x + 5 \\ \underline{x^2 \;\; + x - 1} \\ 5x + 4 \end{array}$$

  we find that $x^3 + 3x + 5 = (x-1)(x^2 + x - 1) + 5x + 4$ and, equivalently, dividing both sides by $x^2 + x - 1$, we have

$$\frac{x^3 + 3x + 5}{x^2 + x - 1} = x - 1 + \frac{5x + 4}{x^2 + x - 1}.$$